Maple 2018.2 Integration Test Results
on the problems in "1 Algebraic functions/1.2 Trinomial products/1.2.2 Quartic"

Test results for the 297 problems in "1.2.2.2 (d x)^m (a+b x^2+c x^4)^p.txt"

Problem 2: Unable to integrate problem.

$$\int \frac{1}{\left(b^{2} x^{4}+2 a b x^{2}+a^{2}\right)^{1/4}}\, dx$$

Optimal(type 3, 48 leaves, 2 steps):

$$\frac{\operatorname{arcsinh}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right) \sqrt{a}\, \sqrt{1+\frac{b x^{2}}{a}}}{\left(b^{2} x^{4}+2 a b x^{2}+a^{2}\right)^{1/4} \sqrt{b}}$$

Result(type 8, 22 leaves):

$$\int \frac{1}{\left(b^{2} x^{4}+2 a b x^{2}+a^{2}\right)^{1/4}}\, dx$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^{4}+2 a x^{2}+a^{2}+1}\, dx$$

Optimal(type 3, 223 leaves, 9 steps):

$$-\frac{\ln\left(x^{2}+\sqrt{a^{2}+1}-x \sqrt{2}\, \sqrt{-a+\sqrt{a^{2}+1}}\right) \sqrt{2}}{8 \sqrt{a^{2}+1}\, \sqrt{-a+\sqrt{a^{2}+1}}}+\frac{\ln\left(x^{2}+\sqrt{a^{2}+1}+x \sqrt{2}\, \sqrt{-a+\sqrt{a^{2}+1}}\right) \sqrt{2}}{8 \sqrt{a^{2}+1}\, \sqrt{-a+\sqrt{a^{2}+1}}}-\frac{\arctan\left(\frac{-x \sqrt{2}+\sqrt{-a+\sqrt{a^{2}+1}}}{\sqrt{a+\sqrt{a^{2}+1}}}\right) \sqrt{2}}{4 \sqrt{a^{2}+1}\, \sqrt{a+\sqrt{a^{2}+1}}}$$
$$+\frac{\arctan\left(\frac{x \sqrt{2}+\sqrt{-a+\sqrt{a^{2}+1}}}{\sqrt{a+\sqrt{a^{2}+1}}}\right) \sqrt{2}}{4 \sqrt{a^{2}+1}\, \sqrt{a+\sqrt{a^{2}+1}}}$$

Result(type 3, 1069 leaves):

$$-\frac{\ln\left(x^{2}-\sqrt{2 \sqrt{a^{2}+1}-2 a}\, x+\sqrt{a^{2}+1}\right) \sqrt{2 \sqrt{a^{2}+1}-2 a}\, a^{2}}{8 \left(a^{2}+1\right)}-\frac{\ln\left(x^{2}-\sqrt{2 \sqrt{a^{2}+1}-2 a}\, x+\sqrt{a^{2}+1}\right) \sqrt{2 \sqrt{a^{2}+1}-2 a}\, a^{3}}{8 \left(a^{2}+1\right)^{3/2}}$$
$$-\frac{\ln\left(x^{2}-\sqrt{2 \sqrt{a^{2}+1}-2 a}\, x+\sqrt{a^{2}+1}\right) \sqrt{2 \sqrt{a^{2}+1}-2 a}}{8 \left(a^{2}+1\right)}-\frac{\ln\left(x^{2}-\sqrt{2 \sqrt{a^{2}+1}-2 a}\, x+\sqrt{a^{2}+1}\right) \sqrt{2 \sqrt{a^{2}+1}-2 a}\, a}{8 \left(a^{2}+1\right)^{3/2}}$$

$$- \frac{\arctan\left(\frac{2x - \sqrt{2\sqrt{a^2+1} - 2a}}{\sqrt{2\sqrt{a^2+1}+2a}}\right) a^2}{2\sqrt{a^2+1}\sqrt{2\sqrt{a^2+1}+2a}} + \frac{\arctan\left(\frac{2x - \sqrt{2\sqrt{a^2+1} - 2a}}{\sqrt{2\sqrt{a^2+1}+2a}}\right) a^4}{2(a^2+1)^{3/2}\sqrt{2\sqrt{a^2+1}+2a}} - \frac{\arctan\left(\frac{2x - \sqrt{2\sqrt{a^2+1} - 2a}}{\sqrt{2\sqrt{a^2+1}+2a}}\right)}{2\sqrt{a^2+1}\sqrt{2\sqrt{a^2+1}+2a}}$$

$$+ \frac{3\arctan\left(\frac{2x - \sqrt{2\sqrt{a^2+1} - 2a}}{\sqrt{2\sqrt{a^2+1}+2a}}\right) a^2}{2(a^2+1)^{3/2}\sqrt{2\sqrt{a^2+1}+2a}} + \frac{\arctan\left(\frac{2x - \sqrt{2\sqrt{a^2+1} - 2a}}{\sqrt{2\sqrt{a^2+1}+2a}}\right)}{(a^2+1)^{3/2}\sqrt{2\sqrt{a^2+1}+2a}} + \frac{\ln\left(x^2 + \sqrt{2\sqrt{a^2+1}-2a}\, x + \sqrt{a^2+1}\right)\sqrt{2\sqrt{a^2+1}-2a}\, a^2}{8(a^2+1)}$$

$$+ \frac{\ln\left(x^2 + \sqrt{2\sqrt{a^2+1}-2a}\, x + \sqrt{a^2+1}\right)\sqrt{2\sqrt{a^2+1}-2a}\, a^3}{8(a^2+1)^{3/2}} + \frac{\ln\left(x^2 + \sqrt{2\sqrt{a^2+1}-2a}\, x + \sqrt{a^2+1}\right)\sqrt{2\sqrt{a^2+1}-2a}}{8(a^2+1)}$$

$$+ \frac{\ln\left(x^2 + \sqrt{2\sqrt{a^2+1}-2a}\, x + \sqrt{a^2+1}\right)\sqrt{2\sqrt{a^2+1}-2a}\, a}{8(a^2+1)^{3/2}} - \frac{\arctan\left(\frac{2x + \sqrt{2\sqrt{a^2+1} - 2a}}{\sqrt{2\sqrt{a^2+1}+2a}}\right) a^2}{2\sqrt{a^2+1}\sqrt{2\sqrt{a^2+1}+2a}} + \frac{\arctan\left(\frac{2x + \sqrt{2\sqrt{a^2+1} - 2a}}{\sqrt{2\sqrt{a^2+1}+2a}}\right) a^4}{2(a^2+1)^{3/2}\sqrt{2\sqrt{a^2+1}+2a}}$$

$$- \frac{\arctan\left(\frac{2x + \sqrt{2\sqrt{a^2+1} - 2a}}{\sqrt{2\sqrt{a^2+1}+2a}}\right)}{2\sqrt{a^2+1}\sqrt{2\sqrt{a^2+1}+2a}} + \frac{3\arctan\left(\frac{2x + \sqrt{2\sqrt{a^2+1} - 2a}}{\sqrt{2\sqrt{a^2+1}+2a}}\right) a^2}{2(a^2+1)^{3/2}\sqrt{2\sqrt{a^2+1}+2a}} + \frac{\arctan\left(\frac{2x + \sqrt{2\sqrt{a^2+1} - 2a}}{\sqrt{2\sqrt{a^2+1}+2a}}\right)}{(a^2+1)^{3/2}\sqrt{2\sqrt{a^2+1}+2a}}$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4 + 2x^2 + 2}\, dx$$

Optimal(type 3, 124 leaves, 9 steps):

$$-\frac{\ln\left(x^2 + \sqrt{2} - x\sqrt{-2+2\sqrt{2}}\right)}{8\sqrt{\sqrt{2}-1}} + \frac{\ln\left(x^2 + \sqrt{2} + x\sqrt{-2+2\sqrt{2}}\right)}{8\sqrt{\sqrt{2}-1}} - \frac{\arctan\left(\frac{-2x + \sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)\sqrt{\sqrt{2}-1}}{4}$$

$$+ \frac{\arctan\left(\frac{2x + \sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)\sqrt{\sqrt{2}-1}}{4}$$

Result(type 3, 385 leaves):

$$\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{-2+2\sqrt{2}}\right)\sqrt{-2+2\sqrt{2}}\,\sqrt{2}}{16}+\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{-2+2\sqrt{2}}\right)\sqrt{-2+2\sqrt{2}}}{8}-\frac{\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)\left(-2+2\sqrt{2}\right)\sqrt{2}}{8\sqrt{2+2\sqrt{2}}}$$

$$-\frac{\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)\left(-2+2\sqrt{2}\right)}{4\sqrt{2+2\sqrt{2}}}+\frac{\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)\sqrt{2}}{2\sqrt{2+2\sqrt{2}}}-\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{-2+2\sqrt{2}}\right)\sqrt{-2+2\sqrt{2}}\,\sqrt{2}}{16}$$

$$-\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{-2+2\sqrt{2}}\right)\sqrt{-2+2\sqrt{2}}}{8}-\frac{\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)\left(-2+2\sqrt{2}\right)\sqrt{2}}{8\sqrt{2+2\sqrt{2}}}-\frac{\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)\left(-2+2\sqrt{2}\right)}{4\sqrt{2+2\sqrt{2}}}$$

$$+\frac{\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)\sqrt{2}}{2\sqrt{2+2\sqrt{2}}}$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{-3x^4+5x^2+2}}\,\mathrm{d}x$$

Optimal(type 4, 13 leaves, 2 steps):

$$\operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2},\mathrm{I}\sqrt{6}\right)$$

Result(type 4, 50 leaves):

$$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}}{2},\mathrm{I}\sqrt{6}\right)}{2\sqrt{-3x^4+5x^2+2}}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{-3x^4+4x^2+2}}\,\mathrm{d}x$$

Optimal(type 4, 37 leaves, 2 steps):

$$\frac{\mathrm{EllipticF}\left(\frac{x\sqrt{-4+2\sqrt{10}}}{2}, \frac{I\sqrt{6}}{3}+\frac{I\sqrt{15}}{3}\right)\sqrt{12+6\sqrt{10}}}{6}$$

Result(type 4, 83 leaves):

$$\frac{2\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^2}\;\mathrm{EllipticF}\left(\frac{x\sqrt{-4+2\sqrt{10}}}{2}, \frac{I\sqrt{6}}{3}+\frac{I\sqrt{15}}{3}\right)}{\sqrt{-4+2\sqrt{10}}\,\sqrt{-3x^4+4x^2+2}}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-3x^4+3x^2+2}}\, dx$$

Optimal(type 4, 37 leaves, 2 steps):

$$\frac{\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{\sqrt{3+\sqrt{33}}}, \frac{I\sqrt{6}}{4}+\frac{I\sqrt{22}}{4}\right)\sqrt{2}}{\sqrt{-3+\sqrt{33}}}$$

Result(type 4, 79 leaves):

$$\frac{2\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\;\mathrm{EllipticF}\left(\frac{x\sqrt{-3+\sqrt{33}}}{2}, \frac{I\sqrt{6}}{4}+\frac{I\sqrt{22}}{4}\right)}{\sqrt{-3+\sqrt{33}}\,\sqrt{-3x^4+3x^2+2}}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-3x^4+2x^2+2}}\, dx$$

Optimal(type 4, 34 leaves, 2 steps):

$$\frac{\mathrm{EllipticF}\left(\frac{x\sqrt{3}}{\sqrt{1+\sqrt{7}}}, \frac{I\sqrt{6}}{6}+\frac{I\sqrt{42}}{6}\right)}{\sqrt{-1+\sqrt{7}}}$$

Result(type 4, 83 leaves):

$$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{\sqrt{7}}{2}-\frac{1}{2}\right)x^2}\;\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2\sqrt{7}}}{2}, \frac{I\sqrt{6}}{6}+\frac{I\sqrt{42}}{6}\right)}{\sqrt{-2+2\sqrt{7}}\,\sqrt{-3x^4+2x^2+2}}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-3x^4-2x^2+2}}\, dx$$

Optimal(type 4, 34 leaves, 2 steps):

$$\frac{\text{EllipticF}\left(\frac{x\sqrt{3}}{\sqrt{-1+\sqrt{7}}}, \frac{I\sqrt{42}}{6}-\frac{I\sqrt{6}}{6}\right)}{\sqrt{1+\sqrt{7}}}$$

Result(type 4, 83 leaves):

$$\frac{2\sqrt{1-\left(\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\ \text{EllipticF}\left(\frac{\sqrt{2+2\sqrt{7}}\,x}{2}, \frac{I\sqrt{42}}{6}-\frac{I\sqrt{6}}{6}\right)}{\sqrt{2+2\sqrt{7}}\,\sqrt{-3x^4-2x^2+2}}$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-2x^4+5x^2+3}}\, dx$$

Optimal(type 4, 13 leaves, 2 steps):

$$\text{EllipticF}\left(\frac{x\sqrt{3}}{3}, I\sqrt{6}\right)$$

Result(type 4, 50 leaves):

$$\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{2x^2+1}\ \text{EllipticF}\left(\frac{x\sqrt{3}}{3}, I\sqrt{6}\right)}{3\sqrt{-2x^4+5x^2+3}}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-2x^4-x^2+3}}\, dx$$

Optimal(type 4, 13 leaves, 2 steps):

$$\frac{\text{EllipticF}\left(x, \frac{I}{3}\sqrt{6}\right)\sqrt{3}}{3}$$

Result(type 4, 42 leaves):

$$\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+9}\,\mathrm{EllipticF}\left(x, \frac{I}{3}\sqrt{6}\right)}{3\sqrt{-2x^4-x^2+3}}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-2x^4-5x^2+3}}\,dx$$

Optimal(type 4, 17 leaves, 2 steps):

$$\frac{\mathrm{EllipticF}\left(x\sqrt{2}, \frac{I}{6}\sqrt{6}\right)\sqrt{6}}{6}$$

Result(type 4, 49 leaves):

$$\frac{\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x\sqrt{2}, \frac{I}{6}\sqrt{6}\right)}{6\sqrt{-2x^4-5x^2+3}}$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-2x^4-7x^2+3}}\,dx$$

Optimal(type 4, 35 leaves, 2 steps):

$$\frac{\mathrm{EllipticF}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}, \frac{I\sqrt{438}}{12}-\frac{7I\sqrt{6}}{12}\right)\sqrt{2}}{\sqrt{7+\sqrt{73}}}$$

Result(type 4, 83 leaves):

$$\frac{6\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{42+6\sqrt{73}}}{6}, \frac{I\sqrt{438}}{12}-\frac{7I\sqrt{6}}{12}\right)}{\sqrt{42+6\sqrt{73}}\,\sqrt{-2x^4-7x^2+3}}$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-2x^4+5x^2+2}}\,dx$$

Optimal(type 4, 31 leaves, 2 steps):

$$\frac{\mathrm{EllipticF}\left(\frac{2x}{\sqrt{5+\sqrt{41}}}, \frac{5I}{4}+\frac{I\sqrt{41}}{4}\right)\sqrt{2}}{\sqrt{-5+\sqrt{41}}}$$

Result(type 4, 75 leaves):

$$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{41}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{41}}{4}\right)x^2}\;\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{41}}}{2}, \frac{5I}{4}+\frac{I\sqrt{41}}{4}\right)}{\sqrt{-5+\sqrt{41}}\,\sqrt{-2x^4+5x^2+2}}$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-7x^4+5x^2+2}}\,dx$$

Optimal(type 4, 13 leaves, 2 steps):

$$\frac{\mathrm{EllipticF}\left(x, \frac{I}{2}\sqrt{14}\right)\sqrt{2}}{2}$$

Result(type 4, 42 leaves):

$$\frac{\sqrt{-x^2+1}\,\sqrt{14x^2+4}\;\mathrm{EllipticF}\left(x, \frac{I}{2}\sqrt{14}\right)}{2\sqrt{-7x^4+5x^2+2}}$$

Problem 40: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-9x^4+5x^2+2}}\,dx$$

Optimal(type 4, 38 leaves, 2 steps):

$$\frac{\mathrm{EllipticF}\left(\frac{3x\sqrt{2}}{\sqrt{5+\sqrt{97}}}, \frac{5I\sqrt{2}}{12}+\frac{I\sqrt{194}}{12}\right)\sqrt{2}}{\sqrt{-5+\sqrt{97}}}$$

Result(type 4, 79 leaves):

$$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{97}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{97}}{4}\right)x^2}\;\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{97}}}{2}, \frac{5I\sqrt{2}}{12}+\frac{I\sqrt{194}}{12}\right)}{\sqrt{-5+\sqrt{97}}\,\sqrt{-9x^4+5x^2+2}}$$

Problem 67: Result more than twice size of optimal antiderivative.

$$\int \frac{(c x^4 + b x^2)^{3/2}}{x^7}\, dx$$

Optimal(type 3, 63 leaves, 5 steps):

$$-\frac{(c x^4 + b x^2)^{3/2}}{3 x^6} + c^{3/2} \operatorname{arctanh}\left(\frac{x^2 \sqrt{c}}{\sqrt{c x^4 + b x^2}}\right) - \frac{c \sqrt{c x^4 + b x^2}}{x^2}$$

Result(type 3, 128 leaves):

$$\frac{(c x^4 + b x^2)^{3/2} \left(2 c^{5/2} (c x^2 + b)^{3/2} x^4 + 3 c^{5/2} \sqrt{c x^2 + b}\, x^4 b - 2 c^{3/2} (c x^2 + b)^{5/2} x^2 + 3 \ln\left(x \sqrt{c} + \sqrt{c x^2 + b}\right) x^3 b^2 c^2 - (c x^2 + b)^{5/2} b \sqrt{c}\right)}{3 x^6 (c x^2 + b)^{3/2} b^2 \sqrt{c}}$$

Problem 108: Result more than twice size of optimal antiderivative.

$$\int (c x)^m (c x^4 + b x^2)^3\, dx$$

Optimal(type 3, 73 leaves, 4 steps):

$$\frac{b^3 x^7 (c x)^m}{7 + m} + \frac{3 b^2 c x^9 (c x)^m}{9 + m} + \frac{3 b c^2 x^{11} (c x)^m}{11 + m} + \frac{c^3 x^{13} (c x)^m}{13 + m}$$

Result(type 3, 180 leaves):

$$\frac{1}{(13 + m) (11 + m) (9 + m) (7 + m)} \left((c x)^m (c^3 m^3 x^6 + 27 c^3 m^2 x^6 + 3 b c^2 m^3 x^4 + 239 c^3 m x^6 + 87 b c^2 m^2 x^4 + 693 c^3 x^6 + 3 b^2 c m^3 x^2 + 813 b c^2 m x^4 + 93 b^2 c m^2 x^2 + 2457 c^2 x^4 b + b^3 m^3 + 933 b^2 c m x^2 + 33 b^3 m^2 + 3003 c x^2 b^2 + 359 b^3 m + 1287 b^3) x^7\right)$$

Problem 109: Unable to integrate problem.

$$\int \frac{(c x)^m}{(c x^4 + b x^2)^2}\, dx$$

Optimal(type 5, 43 leaves, 3 steps):

$$-\frac{(c x)^m \operatorname{hypergeom}\left(\left[2, -\frac{3}{2} + \frac{m}{2}\right], \left[-\frac{1}{2} + \frac{m}{2}\right], -\frac{c x^2}{b}\right)}{b^2 (3 - m) x^3}$$

Result(type 8, 21 leaves):

$$\int \frac{(c x)^m}{(c x^4 + b x^2)^2}\, dx$$

Problem 127: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(b^2 x^4 + 2 a b x^2 + a^2\right)^3}{x^{15}}\, dx$$

Optimal(type 1, 17 leaves, 2 steps):

$$-\frac{\left(b x^2 + a\right)^7}{14 a x^{14}}$$

Result(type 1, 68 leaves):

$$-\frac{b^6}{2 x^2} - \frac{3 a b^5}{2 x^4} - \frac{5 a^2 b^4}{2 x^6} - \frac{5 a^3 b^3}{2 x^8} - \frac{a^6}{14 x^{14}} - \frac{a^5 b}{2 x^{12}} - \frac{3 a^4 b^2}{2 x^{10}}$$

Problem 158: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(b^2 x^4 + 2 a b x^2 + a^2\right)^{5/2}}{x^{13}}\, dx$$

Optimal(type 2, 28 leaves, 3 steps):

$$-\frac{\left(b x^2 + a\right)^5 \sqrt{\left(b x^2 + a\right)^2}}{12 a x^{12}}$$

Result(type 2, 77 leaves):

$$-\frac{\left(6 b^5 x^{10} + 15 a b^4 x^8 + 20 a^2 b^3 x^6 + 15 a^3 b^2 x^4 + 6 a^4 b x^2 + a^5\right) \left(\left(b x^2 + a\right)^2\right)^{5/2}}{12 x^{12} \left(b x^2 + a\right)^5}$$

Problem 177: Unable to integrate problem.

$$\int \frac{x^2}{\left(b^2 x^4 + 2 a b x^2 + a^2\right)^{2/3}}\, dx$$

Optimal(type 4, 524 leaves, 6 steps):

$$-\frac{3 x \left(b x^2 + a\right)}{2 b \left(b^2 x^4 + 2 a b x^2 + a^2\right)^{2/3}} - \frac{9 a x \left(1 + \frac{b x^2}{a}\right)^{4/3}}{2 b \left(b^2 x^4 + 2 a b x^2 + a^2\right)^{2/3} \left(1 - \left(1 + \frac{b x^2}{a}\right)^{1/3} - \sqrt{3}\right)}$$

$$- \frac{1}{2 b^2 x \left(b^2 x^4 + 2 a b x^2 + a^2\right)^{2/3} \sqrt{\frac{-1 + \left(1 + \frac{b x^2}{a}\right)^{1/3}}{\left(1 - \left(1 + \frac{b x^2}{a}\right)^{1/3} - \sqrt{3}\right)^2}}} \Bigg(3\, 3^{3/4} a^2 \left(1 + \frac{b x^2}{a}\right)^{4/3} \left(1 - \left(1 + \frac{b x^2}{a}\right)^{1/}\right.$$

$$\left.{}^{3}\right)\text{EllipticF}\left(\frac{1-\left(1+\frac{b x^2}{a}\right)^{1/3}+\sqrt{3}}{1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}}, 2\,\mathrm{I}-\mathrm{I}\sqrt{3}\right)\sqrt{\frac{1+\left(1+\frac{b x^2}{a}\right)^{1/3}+\left(1+\frac{b x^2}{a}\right)^{2/3}}{\left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}\right)^2}}\,\sqrt{2}\Bigg)$$

$$+\frac{1}{4 b^2 x \left(b^2 x^4+2 a b x^2+a^2\right)^{2/3}\sqrt{\frac{-1+\left(1+\frac{b x^2}{a}\right)^{1/3}}{\left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}\right)^2}}}\Bigg(9\, 3^{1/4} a^2 \left(1+\frac{b x^2}{a}\right)^{4/3}\left(1-\left(1+\frac{b x^2}{a}\right)^{1/}\right.$$

$$\left.{}^{3}\right)\text{EllipticE}\left(\frac{1-\left(1+\frac{b x^2}{a}\right)^{1/3}+\sqrt{3}}{1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}}, 2\,\mathrm{I}-\mathrm{I}\sqrt{3}\right)\sqrt{\frac{1+\left(1+\frac{b x^2}{a}\right)^{1/3}+\left(1+\frac{b x^2}{a}\right)^{2/3}}{\left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}\right)^2}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\Bigg)$$

Result(type 8, 26 leaves):

$$\int \frac{x^2}{\left(b^2 x^4+2 a b x^2+a^2\right)^{2/3}}\, dx$$

Problem 178: Unable to integrate problem.

$$\int \frac{1}{\left(b^2 x^4+2 a b x^2+a^2\right)^{2/3}}\, dx$$

Optimal(type 4, 516 leaves, 6 steps):

$$\frac{3 x \left(b x^2+a\right)}{2 a \left(b^2 x^4+2 a b x^2+a^2\right)^{2/3}}+\frac{3 x \left(1+\frac{b x^2}{a}\right)^{4/3}}{2 \left(b^2 x^4+2 a b x^2+a^2\right)^{2/3}\left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}\right)}$$

$$+\frac{1}{2 b x \left(b^2 x^4+2 a b x^2+a^2\right)^{2/3}\sqrt{\frac{-1+\left(1+\frac{b x^2}{a}\right)^{1/3}}{\left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}\right)^2}}}\Bigg(3^{3/4} a \left(1+\frac{b x^2}{a}\right)^{4/3}\left(1-\left(1+\frac{b x^2}{a}\right)^{1/}\right.$$

$$\left.{}^{3}\right)\mathrm{EllipticF}\left(\frac{1-\left(1+\frac{b x^2}{a}\right)^{1/3}+\sqrt{3}}{1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}}, 2\,\mathrm{I}-\mathrm{I}\sqrt{3}\right)\sqrt{\frac{1+\left(1+\frac{b x^2}{a}\right)^{1/3}+\left(1+\frac{b x^2}{a}\right)^{2/3}}{\left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}\right)^2}}\,\sqrt{2}\Bigg)$$

$$-\frac{1}{4 b x \left(b^2 x^4+2 a b x^2+a^2\right)^{2/3}\sqrt{\frac{-1+\left(1+\frac{b x^2}{a}\right)^{1/3}}{\left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}\right)^2}}}\Bigg(3\, 3^{1/4} a \left(1+\frac{b x^2}{a}\right)^{4/3}\left(1-\left(1+\frac{b x^2}{a}\right)^{1/}\right.$$

$$\left.{}^{3}\right)\mathrm{EllipticE}\left(\frac{1-\left(1+\frac{b x^2}{a}\right)^{1/3}+\sqrt{3}}{1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}}, 2\,\mathrm{I}-\mathrm{I}\sqrt{3}\right)\sqrt{\frac{1+\left(1+\frac{b x^2}{a}\right)^{1/3}+\left(1+\frac{b x^2}{a}\right)^{2/3}}{\left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}\right)^2}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\Bigg)$$

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Result(type 8, 22 leaves):
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$$\int \frac{1}{\left(b^2 x^4+2 a b x^2+a^2\right)^{2/3}}\,dx$$

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Problem 179: Unable to integrate problem.
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$$\int \frac{1}{x^2\left(b^2 x^4+2 a b x^2+a^2\right)^{2/3}}\,dx$$

```
Optimal(type 4, 551 leaves, 7 steps):
```

$$\frac{3\left(b x^2+a\right)}{2 a x\left(b^2 x^4+2 a b x^2+a^2\right)^{2/3}}-\frac{5\left(b x^2+a\right)^2}{2 a^2 x\left(b^2 x^4+2 a b x^2+a^2\right)^{2/3}}-\frac{5 b x\left(1+\frac{b x^2}{a}\right)^{4/3}}{2 a\left(b^2 x^4+2 a b x^2+a^2\right)^{2/3}\left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}\right)}$$

$$-\frac{1}{6 x\left(b^2 x^4+2 a b x^2+a^2\right)^{2/3}\sqrt{\frac{-1+\left(1+\frac{b x^2}{a}\right)^{1/3}}{\left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}\right)^2}}}\Bigg(5\left(1+\frac{b x^2}{a}\right)^{4/3}\left(1-\left(1+\frac{b x^2}{a}\right)^{1/}\right.$$

$$3\Big)\,\mathrm{EllipticF}\left(\frac{1-\left(1+\frac{b x^2}{a}\right)^{1/3}+\sqrt{3}}{1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}}, 2 I - I\sqrt{3}\right)\sqrt{\frac{1+\left(1+\frac{b x^2}{a}\right)^{1/3}+\left(1+\frac{b x^2}{a}\right)^{2/3}}{\left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}\right)^2}}\; 3^{3/4}\sqrt{2}\Bigg)$$

$$+\frac{1}{4 x\left(b^2 x^4+2 a b x^2+a^2\right)^{2/3}\sqrt{\frac{-1+\left(1+\frac{b x^2}{a}\right)^{1/3}}{\left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}\right)^2}}}\Bigg(5\, 3^{1/4}\left(1+\frac{b x^2}{a}\right)^{4/3}\left(1-\left(1+\frac{b x^2}{a}\right)^{1/}\right.$$

$$3\Big)\,\mathrm{EllipticE}\left(\frac{1-\left(1+\frac{b x^2}{a}\right)^{1/3}+\sqrt{3}}{1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}}, 2 I - I\sqrt{3}\right)\sqrt{\frac{1+\left(1+\frac{b x^2}{a}\right)^{1/3}+\left(1+\frac{b x^2}{a}\right)^{2/3}}{\left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}-\sqrt{3}\right)^2}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\Bigg)$$

```
Result(type 8, 86 leaves):
```

$$-\frac{\left(b x^2+a\right)^2}{a^2 x\left(\left(b x^2+a\right)^2\right)^{2/3}}+\frac{\left(\int\frac{b x^2-2 a}{3 a^2\left(x^2+\frac{a}{b}\right)\left(b x^2+a\right)^{1/3}}\,dx\right)\left(b x^2+a\right)^{4/3}}{\left(\left(b x^2+a\right)^2\right)^{2/3}}$$

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Problem 208: Result more than twice size of optimal antiderivative.
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$$\int\frac{(d x)^{13/2}}{\left(b^2 x^4+2 a b x^2+a^2\right)^{3/2}}\,dx$$

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Optimal(type 3, 323 leaves, 14 steps):
```

$$-\frac{11 d^3 (d x)^{7/2}}{16 b^2\sqrt{\left(b x^2+a\right)^2}}-\frac{d (d x)^{11/2}}{4 b\left(b x^2+a\right)\sqrt{\left(b x^2+a\right)^2}}+\frac{77 d^5 (d x)^{3/2}\left(b x^2+a\right)}{48 b^3\sqrt{\left(b x^2+a\right)^2}}+\frac{77 a^{3/4} d^{13/2}\left(b x^2+a\right)\arctan\left(1-\frac{b^{1/4}\sqrt{2}\sqrt{d x}}{a^{1/4}\sqrt{d}}\right)\sqrt{2}}{64 b^{15/4}\sqrt{\left(b x^2+a\right)^2}}$$

$$-\frac{77 a^{3/4} d^{13/2}\left(b x^2+a\right)\arctan\left(1+\frac{b^{1/4}\sqrt{2}\sqrt{d x}}{a^{1/4}\sqrt{d}}\right)\sqrt{2}}{64 b^{15/4}\sqrt{\left(b x^2+a\right)^2}}-\frac{77 a^{3/4} d^{13/2}\left(b x^2+a\right)\ln\left(\sqrt{a}\sqrt{d}+x\sqrt{b}\sqrt{d}-a^{1/4} b^{1/4}\sqrt{2}\sqrt{d x}\right)\sqrt{2}}{128 b^{15/4}\sqrt{\left(b x^2+a\right)^2}}$$

$$+\frac{77 a^{3/4} d^{13/2}\left(b x^2+a\right)\ln\left(\sqrt{a}\sqrt{d}+x\sqrt{b}\sqrt{d}+a^{1/4} b^{1/4}\sqrt{2}\sqrt{d x}\right)\sqrt{2}}{128 b^{15/4}\sqrt{\left(b x^2+a\right)^2}}$$

Result(type 3, 669 leaves):

$$-\frac{1}{384\left(\frac{d^2 a}{b}\right)^{1/4} b^4\left(\left(b x^2+a\right)^2\right)^{3/2}}\Bigg(\Bigg(462\arctan\left(\frac{\sqrt{2}\sqrt{d x}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^4 a b^2 d^4+462\arctan\left(\frac{\sqrt{2}\sqrt{d x}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^4 a b^2 d^4$$
$$+231\sqrt{2}\ln\left(\frac{d x-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{d x+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right)x^4 a b^2 d^4-256\left(\frac{d^2 a}{b}\right)^{1/4}(d x)^{3/2}x^4 b^3 d^2$$
$$+924\arctan\left(\frac{\sqrt{2}\sqrt{d x}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^2 a^2 b d^4+924\arctan\left(\frac{\sqrt{2}\sqrt{d x}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^2 a^2 b d^4$$
$$+462\sqrt{2}\ln\left(\frac{d x-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{d x+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right)x^2 a^2 b d^4-456\left(\frac{d^2 a}{b}\right)^{1/4}(d x)^{7/2}a b^2-512\left(\frac{d^2 a}{b}\right)^{1/4}(d x)^{3/2}x^2 a b^2 d^2$$
$$+462\arctan\left(\frac{\sqrt{2}\sqrt{d x}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\,a^3 d^4+462\arctan\left(\frac{\sqrt{2}\sqrt{d x}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\,a^3 d^4$$
$$+231\sqrt{2}\ln\left(\frac{d x-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{d x+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right)a^3 d^4-616\left(\frac{d^2 a}{b}\right)^{1/4}(d x)^{3/2}a^2 b d^2\Bigg)d^3\left(b x^2+a\right)\Bigg)$$

Problem 209: Result more than twice size of optimal antiderivative.

$$\int\frac{(d x)^{9/2}}{\left(b^2 x^4+2 a b x^2+a^2\right)^{3/2}}\,dx$$

Optimal(type 3, 292 leaves, 13 steps):

$$-\frac{7 d^3 (d x)^{3/2}}{16 b^2\sqrt{\left(b x^2+a\right)^2}}-\frac{d\,(d x)^{7/2}}{4 b\left(b x^2+a\right)\sqrt{\left(b x^2+a\right)^2}}-\frac{21 d^{9/2}\left(b x^2+a\right)\arctan\left(1-\frac{b^{1/4}\sqrt{2}\sqrt{d x}}{a^{1/4}\sqrt{d}}\right)\sqrt{2}}{64 a^{1/4} b^{11/4}\sqrt{\left(b x^2+a\right)^2}}$$

$$+\frac{21 d^{9/2} (b x^2+a) \arctan\left(1+\frac{b^{1/4}\sqrt{2}\sqrt{d x}}{a^{1/4}\sqrt{d}}\right)\sqrt{2}}{64 a^{1/4} b^{11/4}\sqrt{(b x^2+a)^2}}+\frac{21 d^{9/2} (b x^2+a) \ln\left(\sqrt{a}\sqrt{d}+x\sqrt{b}\sqrt{d}-a^{1/4} b^{1/4}\sqrt{2}\sqrt{d x}\right)\sqrt{2}}{128 a^{1/4} b^{11/4}\sqrt{(b x^2+a)^2}}$$

$$-\frac{21 d^{9/2} (b x^2+a) \ln\left(\sqrt{a}\sqrt{d}+x\sqrt{b}\sqrt{d}+a^{1/4} b^{1/4}\sqrt{2}\sqrt{d x}\right)\sqrt{2}}{128 a^{1/4} b^{11/4}\sqrt{(b x^2+a)^2}}$$

Result(type 3, 602 leaves):

$$\frac{1}{128\left(\frac{d^2 a}{b}\right)^{1/4} b^3\left((b x^2+a)^2\right)^{3/2}}\Bigg(\Bigg(42\arctan\left(\frac{\sqrt{2}\sqrt{d x}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\, x^4 b^2 d^4+42\arctan\left(\frac{\sqrt{2}\sqrt{d x}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\, x^4 b^2 d^4$$

$$+21\sqrt{2}\ln\left(\frac{d x-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{d x+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right) x^4 b^2 d^4+84\arctan\left(\frac{\sqrt{2}\sqrt{d x}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\, x^2 a b d^4$$

$$+84\arctan\left(\frac{\sqrt{2}\sqrt{d x}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\, x^2 a b d^4+42\sqrt{2}\ln\left(\frac{d x-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{d x+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right) x^2 a b d^4-88\left(\frac{d^2 a}{b}\right)^{1/4}(d x)^{7/2} b^2$$

$$+42\arctan\left(\frac{\sqrt{2}\sqrt{d x}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\, a^2 d^4+42\arctan\left(\frac{\sqrt{2}\sqrt{d x}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\, a^2 d^4$$

$$+21\sqrt{2}\ln\left(\frac{d x-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{d x+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right) a^2 d^4-56\left(\frac{d^2 a}{b}\right)^{1/4}(d x)^{3/2} a b d^2\Bigg) d\,(b x^2+a)\Bigg)$$

Problem 210: Result more than twice size of optimal antiderivative.

$$\int\frac{(d x)^{3/2}}{(b^2 x^4+2 a b x^2+a^2)^{3/2}}\,dx$$

Optimal(type 3, 293 leaves, 13 steps):

$$-\frac{3 d^{3/2} (b x^2+a) \arctan\left(1-\frac{b^{1/4}\sqrt{2}\sqrt{d x}}{a^{1/4}\sqrt{d}}\right)\sqrt{2}}{64 a^{7/4} b^{5/4}\sqrt{(b x^2+a)^2}}+\frac{3 d^{3/2} (b x^2+a) \arctan\left(1+\frac{b^{1/4}\sqrt{2}\sqrt{d x}}{a^{1/4}\sqrt{d}}\right)\sqrt{2}}{64 a^{7/4} b^{5/4}\sqrt{(b x^2+a)^2}}$$

$$-\frac{3 d^{3/2} (b x^2+a)\ln\left(\sqrt{a}\sqrt{d}+x\sqrt{b}\sqrt{d}-a^{1/4} b^{1/4}\sqrt{2}\sqrt{d x}\right)\sqrt{2}}{128 a^{7/4} b^{5/4}\sqrt{(b x^2+a)^2}}+\frac{3 d^{3/2} (b x^2+a)\ln\left(\sqrt{a}\sqrt{d}+x\sqrt{b}\sqrt{d}+a^{1/4} b^{1/4}\sqrt{2}\sqrt{d x}\right)\sqrt{2}}{128 a^{7/4} b^{5/4}\sqrt{(b x^2+a)^2}}$$

$$+\frac{d\sqrt{d x}}{16 a b\sqrt{(b x^2+a)^2}}-\frac{d\sqrt{d x}}{4 b (b x^2+a)\sqrt{(b x^2+a)^2}}$$

Result(type 3, 667 leaves):

$$\frac{1}{128 d b a^2 \left((b x^2+a)^2\right)^{3/2}}\Bigg(\Bigg(6\arctan\left(\frac{\sqrt{2}\sqrt{d x}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\left(\frac{d^2 a}{b}\right)^{1/4} x^4 b^2 d^2$$

$$+6\arctan\left(\frac{\sqrt{2}\sqrt{d x}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\left(\frac{d^2 a}{b}\right)^{1/4} x^4 b^2 d^2+3\sqrt{2}\ln\left(\frac{d x+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{d x-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right)\left(\frac{d^2 a}{b}\right)^{1/4} x^4 b^2 d^2$$

$$+12\arctan\left(\frac{\sqrt{2}\sqrt{d x}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\left(\frac{d^2 a}{b}\right)^{1/4} x^2 a b d^2+12\arctan\left(\frac{\sqrt{2}\sqrt{d x}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\left(\frac{d^2 a}{b}\right)^{1/4} x^2 a b d^2$$

$$+6\sqrt{2}\ln\left(\frac{d x+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{d x-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right)\left(\frac{d^2 a}{b}\right)^{1/4} x^2 a b d^2+6\arctan\left(\frac{\sqrt{2}\sqrt{d x}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\left(\frac{d^2 a}{b}\right)^{1/4} a^2 d^2$$

$$+6\arctan\left(\frac{\sqrt{2}\sqrt{d x}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\left(\frac{d^2 a}{b}\right)^{1/4} a^2 d^2+3\sqrt{2}\ln\left(\frac{d x+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{d x-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right)\left(\frac{d^2 a}{b}\right)^{1/4} a^2 d^2+8 (d x)^{5/2} a b$$

$$-24\sqrt{d x}\, a^2 d^2\Bigg)(b x^2+a)\Bigg)$$

Problem 212: Result more than twice size of optimal antiderivative.

$$\int \frac{(dx)^{17/2}}{(b^2x^4+2abx^2+a^2)^{5/2}}\,dx$$

Optimal (type 3, 358 leaves, 15 steps):

$$-\frac{385 d^7 (dx)^{3/2}}{1024 b^4 \sqrt{(bx^2+a)^2}}-\frac{d (dx)^{15/2}}{8 b (bx^2+a)^3 \sqrt{(bx^2+a)^2}}-\frac{5 d^3 (dx)^{11/2}}{32 b^2 (bx^2+a)^2 \sqrt{(bx^2+a)^2}}-\frac{55 d^5 (dx)^{7/2}}{256 b^3 (bx^2+a) \sqrt{(bx^2+a)^2}}$$
$$-\frac{1155 d^{17/2} (bx^2+a) \arctan\left(1-\frac{b^{1/4}\sqrt{2}\sqrt{dx}}{a^{1/4}\sqrt{d}}\right)\sqrt{2}}{4096 a^{1/4} b^{19/4}\sqrt{(bx^2+a)^2}}+\frac{1155 d^{17/2} (bx^2+a) \arctan\left(1+\frac{b^{1/4}\sqrt{2}\sqrt{dx}}{a^{1/4}\sqrt{d}}\right)\sqrt{2}}{4096 a^{1/4} b^{19/4}\sqrt{(bx^2+a)^2}}$$
$$+\frac{1155 d^{17/2} (bx^2+a) \ln\left(\sqrt{a}\sqrt{d}+x\sqrt{b}\sqrt{d}-a^{1/4}b^{1/4}\sqrt{2}\sqrt{dx}\right)\sqrt{2}}{8192 a^{1/4} b^{19/4}\sqrt{(bx^2+a)^2}}$$
$$-\frac{1155 d^{17/2} (bx^2+a) \ln\left(\sqrt{a}\sqrt{d}+x\sqrt{b}\sqrt{d}+a^{1/4}b^{1/4}\sqrt{2}\sqrt{dx}\right)\sqrt{2}}{8192 a^{1/4} b^{19/4}\sqrt{(bx^2+a)^2}}$$

Result (type 3, 1030 leaves):

$$\frac{1}{8192\left(\frac{d^2a}{b}\right)^{1/4} b^5 \left((bx^2+a)^2\right)^{5/2}}\Bigg(\Bigg(2310\arctan\left(\frac{\sqrt{2}\sqrt{dx}+\left(\frac{d^2a}{b}\right)^{1/4}}{\left(\frac{d^2a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^8b^4d^8+2310\arctan\left(\frac{\sqrt{2}\sqrt{dx}-\left(\frac{d^2a}{b}\right)^{1/4}}{\left(\frac{d^2a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^8b^4d^8$$
$$+1155\sqrt{2}\ln\left(\frac{dx-\left(\frac{d^2a}{b}\right)^{1/4}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2a}{b}}}{dx+\left(\frac{d^2a}{b}\right)^{1/4}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2a}{b}}}\right)x^8b^4d^8+9240\arctan\left(\frac{\sqrt{2}\sqrt{dx}+\left(\frac{d^2a}{b}\right)^{1/4}}{\left(\frac{d^2a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^6ab^3d^8$$
$$+9240\arctan\left(\frac{\sqrt{2}\sqrt{dx}-\left(\frac{d^2a}{b}\right)^{1/4}}{\left(\frac{d^2a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^6ab^3d^8+4620\sqrt{2}\ln\left(\frac{dx-\left(\frac{d^2a}{b}\right)^{1/4}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2a}{b}}}{dx+\left(\frac{d^2a}{b}\right)^{1/4}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2a}{b}}}\right)x^6ab^3d^8$$
$$-7144\left(\frac{d^2a}{b}\right)^{1/4}(dx)^{15/2}b^4+13860\arctan\left(\frac{\sqrt{2}\sqrt{dx}+\left(\frac{d^2a}{b}\right)^{1/4}}{\left(\frac{d^2a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^4a^2b^2d^8+13860\arctan\left(\frac{\sqrt{2}\sqrt{dx}-\left(\frac{d^2a}{b}\right)^{1/4}}{\left(\frac{d^2a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^4a^2b^2d^8$$

$$+6930\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{dx+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right)x^4 a^2 b^2 d^8-14040\left(\frac{d^2 a}{b}\right)^{1/4}(dx)^{11/2} a b^3 d^2$$

$$+9240\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^2 a^3 b d^8+9240\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^2 a^3 b d^8$$

$$+4620\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{dx+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right)x^2 a^3 b d^8-11000\left(\frac{d^2 a}{b}\right)^{1/4}(dx)^{7/2} a^2 b^2 d^4$$

$$+2310\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\,a^4 d^8+2310\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\,a^4 d^8$$

$$\left.+1155\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{dx+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right)a^4 d^8-3080\left(\frac{d^2 a}{b}\right)^{1/4}(dx)^{3/2} a^3 b d^6\right| d\,(b x^2+a)\Big)$$

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Problem 213: Result more than twice size of optimal antiderivative.
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$$\int \frac{(dx)^{11/2}}{(b^2 x^4+2 a b x^2+a^2)^{5/2}}\,dx$$

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Optimal(type 3, 361 leaves, 15 steps):
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$$-\frac{d\,(dx)^{9/2}}{8 b\,(b x^2+a)^3\sqrt{(b x^2+a)^2}}-\frac{3 d^3\,(dx)^{5/2}}{32 b^2\,(b x^2+a)^2\sqrt{(b x^2+a)^2}}-\frac{45 d^{11/2}\,(b x^2+a)\arctan\left(1-\frac{b^{1/4}\sqrt{2}\,\sqrt{dx}}{a^{1/4}\sqrt{d}}\right)\sqrt{2}}{4096\,a^{7/4} b^{13/4}\sqrt{(b x^2+a)^2}}$$

$$+\frac{45 d^{11/2}\,(b x^2+a)\arctan\left(1+\frac{b^{1/4}\sqrt{2}\,\sqrt{dx}}{a^{1/4}\sqrt{d}}\right)\sqrt{2}}{4096\,a^{7/4} b^{13/4}\sqrt{(b x^2+a)^2}}-\frac{45 d^{11/2}\,(b x^2+a)\ln\left(\sqrt{a}\,\sqrt{d}+x\sqrt{b}\,\sqrt{d}-a^{1/4} b^{1/4}\sqrt{2}\,\sqrt{dx}\right)\sqrt{2}}{8192\,a^{7/4} b^{13/4}\sqrt{(b x^2+a)^2}}$$

$$+\frac{45 d^{11/2} (b x^2+a) \ln\left(\sqrt{a}\sqrt{d}+x\sqrt{b}\sqrt{d}+a^{1/4} b^{1/4}\sqrt{2}\sqrt{d x}\right)\sqrt{2}}{8192 a^{7/4} b^{13/4}\sqrt{(b x^2+a)^2}}+\frac{15 d^5\sqrt{d x}}{1024 a b^3\sqrt{(b x^2+a)^2}}-\frac{15 d^5\sqrt{d x}}{256 b^3 (b x^2+a)\sqrt{(b x^2+a)^2}}$$

Result(type 3, 1135 leaves):

$$\frac{1}{8192 d b^3 a^2 \left((b x^2+a)^2\right)^{5/2}}\Bigg(\Bigg(45\left(\frac{d^2 a}{b}\right)^{1/4}\ln\left(\frac{d x+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{d x-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right)\sqrt{2}\, x^8 b^4 d^6$$

$$+90\left(\frac{d^2 a}{b}\right)^{1/4}\arctan\left(\frac{\sqrt{2}\sqrt{d x}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\, x^8 b^4 d^6+90\left(\frac{d^2 a}{b}\right)^{1/4}\arctan\left(\frac{\sqrt{2}\sqrt{d x}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\, x^8 b^4 d^6$$

$$+180\left(\frac{d^2 a}{b}\right)^{1/4}\ln\left(\frac{d x+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{d x-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right)\sqrt{2}\, x^6 a b^3 d^6+360\left(\frac{d^2 a}{b}\right)^{1/4}\arctan\left(\frac{\sqrt{2}\sqrt{d x}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\, x^6 a b^3 d^6$$

$$+360\left(\frac{d^2 a}{b}\right)^{1/4}\arctan\left(\frac{\sqrt{2}\sqrt{d x}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\, x^6 a b^3 d^6+270\left(\frac{d^2 a}{b}\right)^{1/4}\ln\left(\frac{d x+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{d x-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right)\sqrt{2}\, x^4 a^2 b^2 d^6$$

$$+540\left(\frac{d^2 a}{b}\right)^{1/4}\arctan\left(\frac{\sqrt{2}\sqrt{d x}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\, x^4 a^2 b^2 d^6+540\left(\frac{d^2 a}{b}\right)^{1/4}\arctan\left(\frac{\sqrt{2}\sqrt{d x}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\, x^4 a^2 b^2 d^6$$

$$+120 (d x)^{13/2} a b^3+180\left(\frac{d^2 a}{b}\right)^{1/4}\ln\left(\frac{d x+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{d x-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right)\sqrt{2}\, x^2 a^3 b d^6$$

$$+360\left(\frac{d^2 a}{b}\right)^{1/4}\arctan\left(\frac{\sqrt{2}\sqrt{d x}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\, x^2 a^3 b d^6+360\left(\frac{d^2 a}{b}\right)^{1/4}\arctan\left(\frac{\sqrt{2}\sqrt{d x}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{2}\, x^2 a^3 b d^6$$

$$- 1912\,(dx)^{9/2} a^2 b^2 d^2 + 45 \left(\frac{d^2 a}{b}\right)^{1/4} \ln\left(\frac{dx + \left(\frac{d^2 a}{b}\right)^{1/4} \sqrt{dx}\,\sqrt{2} + \sqrt{\frac{d^2 a}{b}}}{dx - \left(\frac{d^2 a}{b}\right)^{1/4} \sqrt{dx}\,\sqrt{2} + \sqrt{\frac{d^2 a}{b}}}\right) \sqrt{2}\, a^4 d^6$$

$$+ 90 \left(\frac{d^2 a}{b}\right)^{1/4} \arctan\left(\frac{\sqrt{2}\,\sqrt{dx} + \left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right) \sqrt{2}\, a^4 d^6 + 90 \left(\frac{d^2 a}{b}\right)^{1/4} \arctan\left(\frac{\sqrt{2}\,\sqrt{dx} - \left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right) \sqrt{2}\, a^4 d^6 - 1368\,(dx)^{5/2} a^3 b\, d^4$$

$$\left. - 360 \sqrt{dx}\, a^4 d^6 \right) (b x^2 + a) \Bigg)$$

Problem 214: Result more than twice size of optimal antiderivative.

$$\int \frac{(dx)^{5/2}}{(b^2 x^4 + 2 a b x^2 + a^2)^{5/2}}\, dx$$

Optimal(type 3, 361 leaves, 15 steps):

$$\frac{45 d\,(dx)^{3/2}}{1024 a^3 b \sqrt{(b x^2 + a)^2}} - \frac{d\,(dx)^{3/2}}{8 b\,(b x^2 + a)^3 \sqrt{(b x^2 + a)^2}} + \frac{d\,(dx)^{3/2}}{32 a b\,(b x^2 + a)^2 \sqrt{(b x^2 + a)^2}} + \frac{9 d\,(dx)^{3/2}}{256 a^2 b\,(b x^2 + a) \sqrt{(b x^2 + a)^2}}$$

$$- \frac{45 d^{5/2} (b x^2 + a) \arctan\left(1 - \frac{b^{1/4} \sqrt{2}\,\sqrt{dx}}{a^{1/4} \sqrt{d}}\right) \sqrt{2}}{4096 a^{13/4} b^{7/4} \sqrt{(b x^2 + a)^2}} + \frac{45 d^{5/2} (b x^2 + a) \arctan\left(1 + \frac{b^{1/4} \sqrt{2}\,\sqrt{dx}}{a^{1/4} \sqrt{d}}\right) \sqrt{2}}{4096 a^{13/4} b^{7/4} \sqrt{(b x^2 + a)^2}}$$

$$+ \frac{45 d^{5/2} (b x^2 + a) \ln\left(\sqrt{a}\,\sqrt{d} + x \sqrt{b}\,\sqrt{d} - a^{1/4} b^{1/4} \sqrt{2}\,\sqrt{dx}\right) \sqrt{2}}{8192 a^{13/4} b^{7/4} \sqrt{(b x^2 + a)^2}} - \frac{45 d^{5/2} (b x^2 + a) \ln\left(\sqrt{a}\,\sqrt{d} + x \sqrt{b}\,\sqrt{d} + a^{1/4} b^{1/4} \sqrt{2}\,\sqrt{dx}\right) \sqrt{2}}{8192 a^{13/4} b^{7/4} \sqrt{(b x^2 + a)^2}}$$

Result(type 3, 1035 leaves):

$$\frac{1}{8192 d^5 \left(\frac{d^2 a}{b}\right)^{1/4} b^2 a^3 \left((b x^2 + a)^2\right)^{5/2}} \Bigg(\left(90 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx} + \left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right) \sqrt{2}\, x^8 b^4 d^8 + 90 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx} - \left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right) \sqrt{2}\, x^8 b^4 d^8 \right.$$

$$+45\sqrt{2}\,\ln\left(\frac{d x-\left(\frac{d^{2} a}{b}\right)^{1/4}\sqrt{d x}\,\sqrt{2}+\sqrt{\frac{d^{2} a}{b}}}{d x+\left(\frac{d^{2} a}{b}\right)^{1/4}\sqrt{d x}\,\sqrt{2}+\sqrt{\frac{d^{2} a}{b}}}\right) x^{8} b^{4} d^{8}+360\left(\frac{d^{2} a}{b}\right)^{1/4}(d x)^{15/2} b^{4}+360\arctan\left(\frac{\sqrt{2}\,\sqrt{d x}+\left(\frac{d^{2} a}{b}\right)^{1/4}}{\left(\frac{d^{2} a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^{6} a b^{3} d^{8}$$

$$+360\arctan\left(\frac{\sqrt{2}\,\sqrt{d x}-\left(\frac{d^{2} a}{b}\right)^{1/4}}{\left(\frac{d^{2} a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^{6} a b^{3} d^{8}+180\sqrt{2}\,\ln\left(\frac{d x-\left(\frac{d^{2} a}{b}\right)^{1/4}\sqrt{d x}\,\sqrt{2}+\sqrt{\frac{d^{2} a}{b}}}{d x+\left(\frac{d^{2} a}{b}\right)^{1/4}\sqrt{d x}\,\sqrt{2}+\sqrt{\frac{d^{2} a}{b}}}\right) x^{6} a b^{3} d^{8}$$

$$+1368\left(\frac{d^{2} a}{b}\right)^{1/4}(d x)^{11/2} a b^{3} d^{2}+540\arctan\left(\frac{\sqrt{2}\,\sqrt{d x}+\left(\frac{d^{2} a}{b}\right)^{1/4}}{\left(\frac{d^{2} a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^{4} a^{2} b^{2} d^{8}+540\arctan\left(\frac{\sqrt{2}\,\sqrt{d x}-\left(\frac{d^{2} a}{b}\right)^{1/4}}{\left(\frac{d^{2} a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^{4} a^{2} b^{2} d^{8}$$

$$+270\sqrt{2}\,\ln\left(\frac{d x-\left(\frac{d^{2} a}{b}\right)^{1/4}\sqrt{d x}\,\sqrt{2}+\sqrt{\frac{d^{2} a}{b}}}{d x+\left(\frac{d^{2} a}{b}\right)^{1/4}\sqrt{d x}\,\sqrt{2}+\sqrt{\frac{d^{2} a}{b}}}\right) x^{4} a^{2} b^{2} d^{8}+1912\left(\frac{d^{2} a}{b}\right)^{1/4}(d x)^{7/2} a^{2} b^{2} d^{4}$$

$$+360\arctan\left(\frac{\sqrt{2}\,\sqrt{d x}+\left(\frac{d^{2} a}{b}\right)^{1/4}}{\left(\frac{d^{2} a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^{2} a^{3} b d^{8}+360\arctan\left(\frac{\sqrt{2}\,\sqrt{d x}-\left(\frac{d^{2} a}{b}\right)^{1/4}}{\left(\frac{d^{2} a}{b}\right)^{1/4}}\right)\sqrt{2}\,x^{2} a^{3} b d^{8}$$

$$+180\sqrt{2}\,\ln\left(\frac{d x-\left(\frac{d^{2} a}{b}\right)^{1/4}\sqrt{d x}\,\sqrt{2}+\sqrt{\frac{d^{2} a}{b}}}{d x+\left(\frac{d^{2} a}{b}\right)^{1/4}\sqrt{d x}\,\sqrt{2}+\sqrt{\frac{d^{2} a}{b}}}\right) x^{2} a^{3} b d^{8}-120\left(\frac{d^{2} a}{b}\right)^{1/4}(d x)^{3/2} a^{3} b d^{6}+90\arctan\left(\frac{\sqrt{2}\,\sqrt{d x}+\left(\frac{d^{2} a}{b}\right)^{1/4}}{\left(\frac{d^{2} a}{b}\right)^{1/4}}\right)\sqrt{2}\,a^{4} d^{8}$$

$$+90\arctan\left(\frac{\sqrt{2}\,\sqrt{d x}-\left(\frac{d^{2} a}{b}\right)^{1/4}}{\left(\frac{d^{2} a}{b}\right)^{1/4}}\right)\sqrt{2}\,a^{4} d^{8}+45\sqrt{2}\,\ln\left(\frac{d x-\left(\frac{d^{2} a}{b}\right)^{1/4}\sqrt{d x}\,\sqrt{2}+\sqrt{\frac{d^{2} a}{b}}}{d x+\left(\frac{d^{2} a}{b}\right)^{1/4}\sqrt{d x}\,\sqrt{2}+\sqrt{\frac{d^{2} a}{b}}}\right) a^{4} d^{8}\Bigg)\left(b x^{2}+a\right)\Bigg)$$

Problem 215: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{(d x)^{3/2}\left(b^{2} x^{4}+2 a b x^{2}+a^{2}\right)^{5/2}}\,dx$$

Optimal(type 3, 391 leaves, 16 steps):

$$\frac{3315\, b^{1/4}\, (b x^2+a) \arctan\left(1-\frac{b^{1/4}\sqrt{2}\,\sqrt{d x}}{a^{1/4}\sqrt{d}}\right)\sqrt{2}}{4096\, a^{21/4}\, d^{3/2}\sqrt{(b x^2+a)^2}} - \frac{3315\, b^{1/4}\, (b x^2+a) \arctan\left(1+\frac{b^{1/4}\sqrt{2}\,\sqrt{d x}}{a^{1/4}\sqrt{d}}\right)\sqrt{2}}{4096\, a^{21/4}\, d^{3/2}\sqrt{(b x^2+a)^2}}$$

$$- \frac{3315\, b^{1/4}\, (b x^2+a) \ln\left(\sqrt{a}\,\sqrt{d}+x\sqrt{b}\,\sqrt{d}-a^{1/4} b^{1/4}\sqrt{2}\,\sqrt{d x}\right)\sqrt{2}}{8192\, a^{21/4}\, d^{3/2}\sqrt{(b x^2+a)^2}}$$

$$+ \frac{3315\, b^{1/4}\, (b x^2+a) \ln\left(\sqrt{a}\,\sqrt{d}+x\sqrt{b}\,\sqrt{d}+a^{1/4} b^{1/4}\sqrt{2}\,\sqrt{d x}\right)\sqrt{2}}{8192\, a^{21/4}\, d^{3/2}\sqrt{(b x^2+a)^2}} + \frac{663}{1024\, a^4\, d\sqrt{d x}\,\sqrt{(b x^2+a)^2}} + \frac{1}{8\, a\, d\, (b x^2+a)^3\sqrt{d x}\,\sqrt{(b x^2+a)^2}}$$

$$+ \frac{17}{96\, a^2\, d\, (b x^2+a)^2\sqrt{d x}\,\sqrt{(b x^2+a)^2}} + \frac{221}{768\, a^3\, d\, (b x^2+a)\sqrt{d x}\,\sqrt{(b x^2+a)^2}} - \frac{3315\, (b x^2+a)}{1024\, a^5\, d\sqrt{d x}\,\sqrt{(b x^2+a)^2}}$$

Result(type 3, 1065 leaves):

$$-\frac{1}{24576\, d\sqrt{d x}\left(\frac{d^2 a}{b}\right)^{1/4} a^5 \left((b x^2+a)^2\right)^{5/2}}\left(\left(19890 \arctan\left(\frac{\sqrt{2}\,\sqrt{d x}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{d x}\,\sqrt{2}\, x^8 b^4\right.\right.$$

$$+ 19890 \arctan\left(\frac{\sqrt{2}\,\sqrt{d x}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{d x}\,\sqrt{2}\, x^8 b^4 + 9945\sqrt{d x}\,\sqrt{2}\ln\left(\frac{d x-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\,\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{d x+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\,\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right) x^8 b^4$$

$$+ 79560\left(\frac{d^2 a}{b}\right)^{1/4} x^8 b^4 + 79560 \arctan\left(\frac{\sqrt{2}\,\sqrt{d x}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{d x}\,\sqrt{2}\, x^6 a b^3 + 79560 \arctan\left(\frac{\sqrt{2}\,\sqrt{d x}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{d x}\,\sqrt{2}\, x^6 a b^3$$

$$+ 39780\sqrt{d x}\,\sqrt{2}\ln\left(\frac{d x-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\,\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{d x+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{d x}\,\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right) x^6 a b^3 + 302328\left(\frac{d^2 a}{b}\right)^{1/4} x^6 a b^3$$

$$+ 119340 \arctan\left(\frac{\sqrt{2}\,\sqrt{d x}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{d x}\,\sqrt{2}\, x^4 a^2 b^2 + 119340 \arctan\left(\frac{\sqrt{2}\,\sqrt{d x}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{d x}\,\sqrt{2}\, x^4 a^2 b^2$$

$$+ 59670\sqrt{dx}\,\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{dx+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right)x^4 a^2 b^2 + 422552\left(\frac{d^2 a}{b}\right)^{1/4} x^4 a^2 b^2$$

$$+ 79560\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{dx}\,\sqrt{2}\,x^2 a^3 b + 79560\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{dx}\,\sqrt{2}\,x^2 a^3 b$$

$$+ 39780\sqrt{dx}\,\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{dx+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right)x^2 a^3 b + 252008\left(\frac{d^2 a}{b}\right)^{1/4} x^2 a^3 b$$

$$+ 19890\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{dx}\,\sqrt{2}\,a^4 + 19890\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{d^2 a}{b}\right)^{1/4}}{\left(\frac{d^2 a}{b}\right)^{1/4}}\right)\sqrt{dx}\,\sqrt{2}\,a^4$$

$$+ 9945\sqrt{dx}\,\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}{dx+\left(\frac{d^2 a}{b}\right)^{1/4}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2 a}{b}}}\right)a^4 + 49152\left(\frac{d^2 a}{b}\right)^{1/4} a^4\Bigg)\left(b x^2+a\right)\Bigg)$$

Problem 217: Unable to integrate problem.

$$\int \frac{(dx)^m}{\left(b^2 x^4+2 a b x^2+a^2\right)^{5/2}}\, dx$$

Optimal(type 5, 60 leaves, 2 steps):

$$\frac{(dx)^{1+m}\left(b x^2+a\right)\operatorname{hypergeom}\left(\left[5, \frac{1}{2}+\frac{m}{2}\right], \left[\frac{3}{2}+\frac{m}{2}\right], -\frac{b x^2}{a}\right)}{a^5 d\,(1+m)\sqrt{\left(b x^2+a\right)^2}}$$

Result(type 8, 28 leaves):

$$\int \frac{(dx)^m}{\left(b^2 x^4+2 a b x^2+a^2\right)^{5/2}}\, dx$$

Problem 218: Unable to integrate problem.

$$\int \frac{(b^2 x^4 + 2 a b x^2 + a^2)^p}{x^3} \, dx$$

Optimal(type 5, 64 leaves, 3 steps):

$$\frac{b\,(b x^2 + a)\,(b^2 x^4 + 2 a b x^2 + a^2)^p \operatorname{hypergeom}\left([2, 1 + 2p], [2 + 2p], 1 + \frac{b x^2}{a}\right)}{2 a^2 (1 + 2p)}$$

Result(type 8, 26 leaves):

$$\int \frac{(b^2 x^4 + 2 a b x^2 + a^2)^p}{x^3} \, dx$$

Problem 219: Unable to integrate problem.

$$\int x^2 (b^2 x^4 + 2 a b x^2 + a^2)^p \, dx$$

Optimal(type 5, 58 leaves, 2 steps):

$$\frac{x^3 (b^2 x^4 + 2 a b x^2 + a^2)^p \operatorname{hypergeom}\left(\left[\frac{3}{2}, -2p\right], \left[\frac{5}{2}\right], -\frac{b x^2}{a}\right)}{3 \left(1 + \frac{b x^2}{a}\right)^{2p}}$$

Result(type 8, 26 leaves):

$$\int x^2 (b^2 x^4 + 2 a b x^2 + a^2)^p \, dx$$

Problem 220: Unable to integrate problem.

$$\int \frac{(b^2 x^4 + 2 a b x^2 + a^2)^p}{x^4} \, dx$$

Optimal(type 5, 58 leaves, 2 steps):

$$-\frac{(b^2 x^4 + 2 a b x^2 + a^2)^p \operatorname{hypergeom}\left(\left[-\frac{3}{2}, -2p\right], \left[-\frac{1}{2}\right], -\frac{b x^2}{a}\right)}{3 x^3 \left(1 + \frac{b x^2}{a}\right)^{2p}}$$

Result(type 8, 26 leaves):

$$\int \frac{(b^2 x^4 + 2 a b x^2 + a^2)^p}{x^4} \, dx$$

Problem 231: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 \left(c x^4 + b x^2 + a\right)^2}\, dx$$

Optimal(type 3, 155 leaves, 8 steps):

$$\frac{3 a c - b^2}{a^2 \left(-4 a c + b^2\right) x^2} + \frac{c x^2 b - 2 a c + b^2}{2 a \left(-4 a c + b^2\right) x^2 \left(c x^4 + b x^2 + a\right)} - \frac{\left(6 a^2 c^2 - 6 a b^2 c + b^4\right) \operatorname{arctanh}\left(\frac{2 c x^2 + b}{\sqrt{-4 a c + b^2}}\right)}{a^3 \left(-4 a c + b^2\right)^{3/2}} - \frac{2 b \ln(x)}{a^3} + \frac{b \ln\left(c x^4 + b x^2 + a\right)}{2 a^3}$$

Result(type 3, 568 leaves):

$$-\frac{c^2 x^2}{a \left(c x^4 + b x^2 + a\right) \left(4 a c - b^2\right)} + \frac{c x^2 b^2}{2 a^2 \left(c x^4 + b x^2 + a\right) \left(4 a c - b^2\right)} - \frac{3 b c}{2 a \left(c x^4 + b x^2 + a\right) \left(4 a c - b^2\right)} + \frac{b^3}{2 a^2 \left(c x^4 + b x^2 + a\right) \left(4 a c - b^2\right)}$$

$$+ \frac{2 c \ln\left(\left(4 a c - b^2\right) \left(c x^4 + b x^2 + a\right)\right) b}{a^2 \left(4 a c - b^2\right)} - \frac{\ln\left(\left(4 a c - b^2\right) \left(c x^4 + b x^2 + a\right)\right) b^3}{2 a^3 \left(4 a c - b^2\right)} - \frac{6 \arctan\left(\frac{2 \left(4 a c - b^2\right) c x^2 + \left(4 a c - b^2\right) b}{\sqrt{64 a^3 c^3 - 48 a^2 b^2 c^2 + 12 a b^4 c - b^6}}\right) c^2}{a \sqrt{64 a^3 c^3 - 48 a^2 b^2 c^2 + 12 a b^4 c - b^6}}$$

$$+ \frac{6 \arctan\left(\frac{2 \left(4 a c - b^2\right) c x^2 + \left(4 a c - b^2\right) b}{\sqrt{64 a^3 c^3 - 48 a^2 b^2 c^2 + 12 a b^4 c - b^6}}\right) b^2 c}{a^2 \sqrt{64 a^3 c^3 - 48 a^2 b^2 c^2 + 12 a b^4 c - b^6}} - \frac{\arctan\left(\frac{2 \left(4 a c - b^2\right) c x^2 + \left(4 a c - b^2\right) b}{\sqrt{64 a^3 c^3 - 48 a^2 b^2 c^2 + 12 a b^4 c - b^6}}\right) b^4}{a^3 \sqrt{64 a^3 c^3 - 48 a^2 b^2 c^2 + 12 a b^4 c - b^6}} - \frac{1}{2 a^2 x^2} - \frac{2 b \ln(x)}{a^3}$$

Problem 232: Result more than twice size of optimal antiderivative.

$$\int \frac{x^8}{\left(c x^4 + b x^2 + a\right)^2}\, dx$$

Optimal(type 3, 284 leaves, 6 steps):

$$\frac{\left(-10 a c + 3 b^2\right) x}{2 c^2 \left(-4 a c + b^2\right)} - \frac{b x^3}{2 c \left(-4 a c + b^2\right)} + \frac{x^5 \left(b x^2 + 2 a\right)}{2 \left(-4 a c + b^2\right) \left(c x^4 + b x^2 + a\right)}$$

$$- \frac{\arctan\left(\frac{x \sqrt{2}\, \sqrt{c}}{\sqrt{b - \sqrt{-4 a c + b^2}}}\right) \left(3 b^3 - 13 a b c + \frac{-20 a^2 c^2 + 19 a b^2 c - 3 b^4}{\sqrt{-4 a c + b^2}}\right) \sqrt{2}}{4 c^{5/2} \left(-4 a c + b^2\right) \sqrt{b - \sqrt{-4 a c + b^2}}}$$

$$- \frac{\arctan\left(\frac{x \sqrt{2}\, \sqrt{c}}{\sqrt{b + \sqrt{-4 a c + b^2}}}\right) \left(3 b^3 - 13 a b c + \frac{20 a^2 c^2 - 19 a b^2 c + 3 b^4}{\sqrt{-4 a c + b^2}}\right) \sqrt{2}}{4 c^{5/2} \left(-4 a c + b^2\right) \sqrt{b + \sqrt{-4 a c + b^2}}}$$

Result(type ?, 2279 leaves): Display of huge result suppressed!

Problem 233: Result more than twice size of optimal antiderivative.

$$\int \frac{x^5}{(c x^4+b x^2+a)^3}\, dx$$

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Optimal(type 3, 122 leaves, 5 steps):
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$$\frac{x^2 (b x^2+2 a)}{4 (-4 a c+b^2) (c x^4+b x^2+a)^2}+\frac{3 a b+(2 a c+b^2) x^2}{2 (-4 a c+b^2)^2 (c x^4+b x^2+a)}-\frac{(2 a c+b^2) \operatorname{arctanh}\left(\frac{2 c x^2+b}{\sqrt{-4 a c+b^2}}\right)}{(-4 a c+b^2)^{5/2}}$$

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Result(type 3, 269 leaves):
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$$\frac{\frac{c (2 a c+b^2) x^6}{16 a^2 c^2-8 a b^2 c+b^4}+\frac{3 b (2 a c+b^2) x^4}{2 (16 a^2 c^2-8 a b^2 c+b^4)}-\frac{a (2 a c-5 b^2) x^2}{16 a^2 c^2-8 a b^2 c+b^4}+\frac{3 a^2 b}{16 a^2 c^2-8 a b^2 c+b^4}}{2 (c x^4+b x^2+a)^2}+\frac{2 \arctan\left(\frac{2 c x^2+b}{\sqrt{4 a c-b^2}}\right) a c}{(16 a^2 c^2-8 a b^2 c+b^4) \sqrt{4 a c-b^2}}$$
$$+\frac{\arctan\left(\frac{2 c x^2+b}{\sqrt{4 a c-b^2}}\right) b^2}{(16 a^2 c^2-8 a b^2 c+b^4) \sqrt{4 a c-b^2}}$$

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Problem 235: Result more than twice size of optimal antiderivative.
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$$\int \frac{1}{x (c x^4+b x^2+a)^3}\, dx$$

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Optimal(type 3, 188 leaves, 9 steps):
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$$\frac{c x^2 b-2 a c+b^2}{4 a (-4 a c+b^2) (c x^4+b x^2+a)^2}+\frac{2 b^4-15 a b^2 c+16 a^2 c^2+2 b c (-7 a c+b^2) x^2}{4 a^2 (-4 a c+b^2)^2 (c x^4+b x^2+a)}+\frac{b (30 a^2 c^2-10 a b^2 c+b^4) \operatorname{arctanh}\left(\frac{2 c x^2+b}{\sqrt{-4 a c+b^2}}\right)}{2 a^3 (-4 a c+b^2)^{5/2}}+\frac{\ln(x)}{a^3}$$
$$-\frac{\ln(c x^4+b x^2+a)}{4 a^3}$$

```
Result(type 3, 1199 leaves):
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$$-\frac{7 c^3 b x^6}{2 a (c x^4+b x^2+a)^2 (16 a^2 c^2-8 a b^2 c+b^4)}+\frac{c^2 b^3 x^6}{2 a^2 (c x^4+b x^2+a)^2 (16 a^2 c^2-8 a b^2 c+b^4)}+\frac{4 c^3 x^4}{(c x^4+b x^2+a)^2 (16 a^2 c^2-8 a b^2 c+b^4)}$$
$$-\frac{29 c^2 x^4 b^2}{4 a (c x^4+b x^2+a)^2 (16 a^2 c^2-8 a b^2 c+b^4)}+\frac{c x^4 b^4}{a^2 (c x^4+b x^2+a)^2 (16 a^2 c^2-8 a b^2 c+b^4)}-\frac{b x^2 c^2}{2 (c x^4+b x^2+a)^2 (16 a^2 c^2-8 a b^2 c+b^4)}$$
$$-\frac{3 b^3 x^2 c}{a (c x^4+b x^2+a)^2 (16 a^2 c^2-8 a b^2 c+b^4)}+\frac{b^5 x^2}{2 a^2 (c x^4+b x^2+a)^2 (16 a^2 c^2-8 a b^2 c+b^4)}+\frac{6 a c^2}{(c x^4+b x^2+a)^2 (16 a^2 c^2-8 a b^2 c+b^4)}$$
$$-\frac{21 b^2 c}{4 (c x^4+b x^2+a)^2 (16 a^2 c^2-8 a b^2 c+b^4)}+\frac{3 b^4}{4 a (c x^4+b x^2+a)^2 (16 a^2 c^2-8 a b^2 c+b^4)}$$

$$-\frac{4c^2\ln\left(\left(16a^2c^2-8ab^2c+b^4\right)\left(cx^4+bx^2+a\right)\right)}{a\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{2c\ln\left(\left(16a^2c^2-8ab^2c+b^4\right)\left(cx^4+bx^2+a\right)\right)b^2}{a^2\left(16a^2c^2-8ab^2c+b^4\right)}$$

$$-\frac{\ln\left(\left(16a^2c^2-8ab^2c+b^4\right)\left(cx^4+bx^2+a\right)\right)b^4}{4a^3\left(16a^2c^2-8ab^2c+b^4\right)}-\frac{15\arctan\left(\frac{2cx^2\left(16a^2c^2-8ab^2c+b^4\right)+b\left(16a^2c^2-8ab^2c+b^4\right)}{\sqrt{1024a^5c^5-1280a^4b^2c^4+640a^3b^4c^3-160a^2b^6c^2+20ab^8c-b^{10}}}\right)bc^2}{a\sqrt{1024a^5c^5-1280a^4b^2c^4+640a^3b^4c^3-160a^2b^6c^2+20ab^8c-b^{10}}}$$

$$+\frac{5\arctan\left(\frac{2cx^2\left(16a^2c^2-8ab^2c+b^4\right)+b\left(16a^2c^2-8ab^2c+b^4\right)}{\sqrt{1024a^5c^5-1280a^4b^2c^4+640a^3b^4c^3-160a^2b^6c^2+20ab^8c-b^{10}}}\right)b^3c}{a^2\sqrt{1024a^5c^5-1280a^4b^2c^4+640a^3b^4c^3-160a^2b^6c^2+20ab^8c-b^{10}}}$$

$$-\frac{\arctan\left(\frac{2cx^2\left(16a^2c^2-8ab^2c+b^4\right)+b\left(16a^2c^2-8ab^2c+b^4\right)}{\sqrt{1024a^5c^5-1280a^4b^2c^4+640a^3b^4c^3-160a^2b^6c^2+20ab^8c-b^{10}}}\right)b^5}{2a^3\sqrt{1024a^5c^5-1280a^4b^2c^4+640a^3b^4c^3-160a^2b^6c^2+20ab^8c-b^{10}}}+\frac{\ln(x)}{a^3}$$

Problem 236: Result more than twice size of optimal antiderivative.

$$\int\frac{x^{10}}{\left(cx^4+bx^2+a\right)^3}\,dx$$

Optimal(type 3, 353 leaves, 7 steps):

$$-\frac{3b\left(-8ac+b^2\right)x}{8c^2\left(-4ac+b^2\right)^2}+\frac{\left(-28ac+b^2\right)x^3}{8c\left(-4ac+b^2\right)^2}+\frac{x^7\left(bx^2+2a\right)}{4\left(-4ac+b^2\right)\left(cx^4+bx^2+a\right)^2}+\frac{x^5\left(12ab-\left(-28ac+b^2\right)x^2\right)}{8\left(-4ac+b^2\right)^2\left(cx^4+bx^2+a\right)}$$

$$+\frac{3\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\left(b^4-9ab^2c+28a^2c^2+\frac{-44a^2bc^2+11ab^3c-b^5}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{16c^{5/2}\left(-4ac+b^2\right)^2\sqrt{b-\sqrt{-4ac+b^2}}}$$

$$+\frac{3\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(b^4-9ab^2c+28a^2c^2+\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{16c^{5/2}\left(-4ac+b^2\right)^2\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type ?, 5424 leaves): Display of huge result suppressed!

Problem 240: Result more than twice size of optimal antiderivative.

$$\int\frac{x^2}{x^4-2x^2+2}\,dx$$

Optimal(type 3, 132 leaves, 9 steps):

$$-\frac{\arctan\left(\frac{-2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)\sqrt{2+2\sqrt{2}}}{4}+\frac{\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)\sqrt{2+2\sqrt{2}}}{4}+\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{2+2\sqrt{2}}\right)}{4\sqrt{2+2\sqrt{2}}}$$
$$-\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{2+2\sqrt{2}}\right)}{4\sqrt{2+2\sqrt{2}}}$$

Result(type 3, 307 leaves):

$$-\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\ln\left(x^2+\sqrt{2}+x\sqrt{2+2\sqrt{2}}\right)}{8}+\frac{\sqrt{2}\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}+\frac{\sqrt{2+2\sqrt{2}}\ln\left(x^2+\sqrt{2}+x\sqrt{2+2\sqrt{2}}\right)}{8}$$
$$-\frac{\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}+\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\ln\left(x^2+\sqrt{2}-x\sqrt{2+2\sqrt{2}}\right)}{8}+\frac{\sqrt{2}\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}$$
$$-\frac{\sqrt{2+2\sqrt{2}}\ln\left(x^2+\sqrt{2}-x\sqrt{2+2\sqrt{2}}\right)}{8}-\frac{\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}$$

Problem 243: Result more than twice size of optimal antiderivative.

$$\int x^5\left(c x^4+b x^2+a\right)^{3/2}\,dx$$

Optimal(type 3, 178 leaves, 7 steps):

$$\frac{\left(-4ac+7b^2\right)\left(2cx^2+b\right)\left(cx^4+bx^2+a\right)^{3/2}}{384c^3}-\frac{7b\left(cx^4+bx^2+a\right)^{5/2}}{120c^2}+\frac{x^2\left(cx^4+bx^2+a\right)^{5/2}}{12c}$$
$$+\frac{\left(-4ac+b^2\right)^2\left(-4ac+7b^2\right)\operatorname{arctanh}\left(\frac{2cx^2+b}{2\sqrt{c}\sqrt{cx^4+bx^2+a}}\right)}{2048c^{9/2}}-\frac{\left(-4ac+b^2\right)\left(-4ac+7b^2\right)\left(2cx^2+b\right)\sqrt{cx^4+bx^2+a}}{1024c^4}$$

Result(type 3, 431 leaves):

$$\frac{a^2x^2\sqrt{cx^4+bx^2+a}}{32c}-\frac{27a^2b\sqrt{cx^4+bx^2+a}}{320c^2}+\frac{9a^2b^2\ln\left(\frac{\frac{b}{2}+cx^2}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{128c^{5/2}}+\frac{cx^{10}\sqrt{cx^4+bx^2+a}}{12}+\frac{13bx^8\sqrt{cx^4+bx^2+a}}{120}$$

$$-\frac{7 b^5 \sqrt{c x^4+b x^2+a}}{1024 c^4}+\frac{7 a x^6 \sqrt{c x^4+b x^2+a}}{48}-\frac{15 b^4 a \ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{512 c^{7/2}}-\frac{9 b^2 a x^2 \sqrt{c x^4+b x^2+a}}{320 c^2}$$
$$+\frac{3 b a x^4 \sqrt{c x^4+b x^2+a}}{160 c}-\frac{a^3 \ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{32 c^{3/2}}+\frac{b^2 x^6 \sqrt{c x^4+b x^2+a}}{320 c}-\frac{7 b^3 x^4 \sqrt{c x^4+b x^2+a}}{1920 c^2}+\frac{7 b^4 x^2 \sqrt{c x^4+b x^2+a}}{1536 c^3}$$
$$+\frac{7 b^6 \ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{2048 c^{9/2}}+\frac{19 b^3 a \sqrt{c x^4+b x^2+a}}{384 c^3}$$

Problem 245: Result more than twice size of optimal antiderivative.

$$\int \frac{(c x^4+b x^2+a)^{3/2}}{x^{11}}\, dx$$

Optimal(type 3, 140 leaves, 6 steps):

$$\frac{b\left(b x^2+2 a\right)\left(c x^4+b x^2+a\right)^{3/2}}{32 a^2 x^8}-\frac{\left(c x^4+b x^2+a\right)^{5/2}}{10 a x^{10}}+\frac{3 b\left(-4 a c+b^2\right)^2 \operatorname{arctanh}\left(\frac{b x^2+2 a}{2 \sqrt{a} \sqrt{c x^4+b x^2+a}}\right)}{512 a^{7/2}}$$
$$-\frac{3 b\left(-4 a c+b^2\right)\left(b x^2+2 a\right) \sqrt{c x^4+b x^2+a}}{256 a^3 x^4}$$

Result(type 3, 336 leaves):

$$-\frac{a \sqrt{c x^4+b x^2+a}}{10 x^{10}}-\frac{11 b \sqrt{c x^4+b x^2+a}}{80 x^8}-\frac{b^2 \sqrt{c x^4+b x^2+a}}{160 a x^6}+\frac{b^3 \sqrt{c x^4+b x^2+a}}{128 a^2 x^4}-\frac{3 b^4 \sqrt{c x^4+b x^2+a}}{256 a^3 x^2}$$
$$+\frac{3 b^5 \ln\left(\frac{2 a+b x^2+2 \sqrt{a} \sqrt{c x^4+b x^2+a}}{x^2}\right)}{512 a^{7/2}}-\frac{3 b^3 c \ln\left(\frac{2 a+b x^2+2 \sqrt{a} \sqrt{c x^4+b x^2+a}}{x^2}\right)}{64 a^{5/2}}+\frac{5 b^2 c \sqrt{c x^4+b x^2+a}}{64 a^2 x^2}-\frac{7 b c \sqrt{c x^4+b x^2+a}}{160 a x^4}$$
$$+\frac{3 b c^2 \ln\left(\frac{2 a+b x^2+2 \sqrt{a} \sqrt{c x^4+b x^2+a}}{x^2}\right)}{32 a^{3/2}}-\frac{c \sqrt{c x^4+b x^2+a}}{5 x^6}-\frac{c^2 \sqrt{c x^4+b x^2+a}}{10 a x^2}$$

Problem 246: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c x^4+b x^2+a\right)^{3/2}}{x^{13}}\, dx$$

Optimal(type 3, 190 leaves, 7 steps):

$$-\frac{\left(-4 a c+7 b^2\right)\left(b x^2+2 a\right)\left(c x^4+b x^2+a\right)^{3/2}}{384 a^3 x^8}-\frac{\left(c x^4+b x^2+a\right)^{5/2}}{12 a x^{12}}+\frac{7 b\left(c x^4+b x^2+a\right)^{5/2}}{120 a^2 x^{10}}$$
$$-\frac{\left(-4 a c+b^2\right)^2\left(-4 a c+7 b^2\right) \operatorname{arctanh}\left(\frac{b x^2+2 a}{2 \sqrt{a} \sqrt{c x^4+b x^2+a}}\right)}{2048 a^{9/2}}+\frac{\left(-4 a c+b^2\right)\left(-4 a c+7 b^2\right)\left(b x^2+2 a\right) \sqrt{c x^4+b x^2+a}}{1024 a^4 x^4}$$

Result(type 3, 456 leaves):

$$-\frac{a \sqrt{c x^4+b x^2+a}}{12 x^{12}}-\frac{13 b \sqrt{c x^4+b x^2+a}}{120 x^{10}}-\frac{7 c \sqrt{c x^4+b x^2+a}}{48 x^8}-\frac{9 b^2 c^2 \ln\left(\frac{2 a+b x^2+2 \sqrt{a} \sqrt{c x^4+b x^2+a}}{x^2}\right)}{128 a^{5/2}}-\frac{3 b c \sqrt{c x^4+b x^2+a}}{160 a x^6}$$
$$+\frac{27 b c^2 \sqrt{c x^4+b x^2+a}}{320 a^2 x^2}+\frac{15 b^4 c \ln\left(\frac{2 a+b x^2+2 \sqrt{a} \sqrt{c x^4+b x^2+a}}{x^2}\right)}{512 a^{7/2}}-\frac{19 b^3 c \sqrt{c x^4+b x^2+a}}{384 a^3 x^2}+\frac{9 b^2 c \sqrt{c x^4+b x^2+a}}{320 a^2 x^4}$$
$$-\frac{7 b^4 \sqrt{c x^4+b x^2+a}}{1536 a^3 x^4}+\frac{7 b^5 \sqrt{c x^4+b x^2+a}}{1024 a^4 x^2}-\frac{7 b^6 \ln\left(\frac{2 a+b x^2+2 \sqrt{a} \sqrt{c x^4+b x^2+a}}{x^2}\right)}{2048 a^{9/2}}-\frac{c^2 \sqrt{c x^4+b x^2+a}}{32 a x^4}$$
$$+\frac{c^3 \ln\left(\frac{2 a+b x^2+2 \sqrt{a} \sqrt{c x^4+b x^2+a}}{x^2}\right)}{32 a^{3/2}}-\frac{b^2 \sqrt{c x^4+b x^2+a}}{320 a x^8}+\frac{7 b^3 \sqrt{c x^4+b x^2+a}}{1920 a^2 x^6}$$

Problem 276: Result is not expressed in closed-form.

$$\int \frac{x^{7/2}}{c x^4+b x^2+a}\, dx$$

Optimal(type 3, 309 leaves, 9 steps):

$$\frac{\arctan\left(\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{-4 a c+b^2}\right)^{1/4}}\right)\left(b+\frac{-2 a c+b^2}{\sqrt{-4 a c+b^2}}\right) 2^{3/4}}{2 c^{5/4}\left(-b-\sqrt{-4 a c+b^2}\right)^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{-4 a c+b^2}\right)^{1/4}}\right)\left(b+\frac{-2 a c+b^2}{\sqrt{-4 a c+b^2}}\right) 2^{3/4}}{2 c^{5/4}\left(-b-\sqrt{-4 a c+b^2}\right)^{3/4}}$$
$$+\frac{\arctan\left(\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{-4 a c+b^2}\right)^{1/4}}\right)\left(b+\frac{2 a c-b^2}{\sqrt{-4 a c+b^2}}\right) 2^{3/4}}{2 c^{5/4}\left(-b+\sqrt{-4 a c+b^2}\right)^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{-4 a c+b^2}\right)^{1/4}}\right)\left(b+\frac{2 a c-b^2}{\sqrt{-4 a c+b^2}}\right) 2^{3/4}}{2 c^{5/4}\left(-b+\sqrt{-4 a c+b^2}\right)^{3/4}}+\frac{2 \sqrt{x}}{c}$$

Result(type 7, 63 leaves):

$$\frac{2\sqrt{x}}{c}+\frac{\sum_{\_R=RootOf(c\_Z^8+b\_Z^4+a)}\frac{(-\_R^4 b-a)\ln(\sqrt{x}-\_R)}{2\_R^7 c+\_R^3 b}}{2c}$$

Problem 277: Result is not expressed in closed-form.

$$\int \frac{1}{\sqrt{x}\,(c x^4+b x^2+a)}\,dx$$

Optimal(type 3, 251 leaves, 8 steps):

$$\frac{2^{3/4}c^{3/4}\arctan\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{-4ac+b^2}\right)^{1/4}}\right)}{\left(-b-\sqrt{-4ac+b^2}\right)^{3/4}\sqrt{-4ac+b^2}}+\frac{2^{3/4}c^{3/4}\operatorname{arctanh}\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{-4ac+b^2}\right)^{1/4}}\right)}{\left(-b-\sqrt{-4ac+b^2}\right)^{3/4}\sqrt{-4ac+b^2}}-\frac{2^{3/4}c^{3/4}\arctan\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{-4ac+b^2}\right)^{1/4}}\right)}{\sqrt{-4ac+b^2}\left(-b+\sqrt{-4ac+b^2}\right)^{3/4}}$$
$$-\frac{2^{3/4}c^{3/4}\operatorname{arctanh}\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{-4ac+b^2}\right)^{1/4}}\right)}{\sqrt{-4ac+b^2}\left(-b+\sqrt{-4ac+b^2}\right)^{3/4}}$$

Result(type 7, 41 leaves):

$$\frac{\left(\sum_{\_R=RootOf(c\_Z^8+b\_Z^4+a)}\frac{\ln(\sqrt{x}-\_R)}{2\_R^7 c+\_R^3 b}\right)}{2}$$

Problem 278: Result is not expressed in closed-form.

$$\int \frac{x^{9/2}}{(c x^4+b x^2+a)^2}\,dx$$

Optimal(type 3, 377 leaves, 9 steps):

$$\frac{x^{3/2}(b x^2+2a)}{2(-4ac+b^2)(c x^4+b x^2+a)}+\frac{\arctan\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{-4ac+b^2}\right)^{1/4}}\right)\left(b+\frac{-12ac-b^2}{\sqrt{-4ac+b^2}}\right)2^{1/4}}{8c^{3/4}(-4ac+b^2)\left(-b+\sqrt{-4ac+b^2}\right)^{1/4}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{-4ac+b^2}\right)^{1/4}}\right)\left(b+\frac{-12ac-b^2}{\sqrt{-4ac+b^2}}\right)2^{1/4}}{8c^{3/4}(-4ac+b^2)\left(-b+\sqrt{-4ac+b^2}\right)^{1/4}}+\frac{\arctan\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{-4ac+b^2}\right)^{1/4}}\right)\left(b^2+12ac+b\sqrt{-4ac+b^2}\right)2^{1/4}}{8c^{3/4}(-4ac+b^2)^{3/2}\left(-b-\sqrt{-4ac+b^2}\right)^{1/4}}$$

$$-\frac{\operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{-4 a c+b^{2}}\right)^{1/4}}\right)\left(b^{2}+12 a c+b \sqrt{-4 a c+b^{2}}\right) 2^{1/4}}{8 c^{3/4}\left(-4 a c+b^{2}\right)^{3/2}\left(-b-\sqrt{-4 a c+b^{2}}\right)^{1/4}}$$

```
Result(type 7, 120 leaves):
```

$$\frac{2\left(-\frac{b x^{7/2}}{4\left(4 a c-b^{2}\right)}-\frac{a x^{3/2}}{2\left(4 a c-b^{2}\right)}\right)}{c x^{4}+b x^{2}+a}+\frac{\left(\sum_{\_R=RootOf\left(c\_Z^{8}+b\_Z^{4}+a\right)} \frac{\left(-\_R^{6} b+6\_R^{2} a\right) \ln\left(\sqrt{x}-\_R\right)}{\left(4 a c-b^{2}\right)\left(2\_R^{7} c+\_R^{3} b\right)}\right)}{8}$$

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Problem 279: Result is not expressed in closed-form.
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$$\int \frac{x^{5/2}}{\left(c x^{4}+b x^{2}+a\right)^{2}} dx$$

```
Optimal(type 3, 350 leaves, 9 steps):
```

$$-\frac{x^{3/2}\left(2 c x^{2}+b\right)}{2\left(-4 a c+b^{2}\right)\left(c x^{4}+b x^{2}+a\right)}+\frac{c^{1/4} \arctan\left(\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{-4 a c+b^{2}}\right)^{1/4}}\right)\left(4 b-\sqrt{-4 a c+b^{2}}\right) 2^{1/4}}{4\left(-4 a c+b^{2}\right)^{3/2}\left(-b+\sqrt{-4 a c+b^{2}}\right)^{1/4}}$$

$$-\frac{c^{1/4} \operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{-4 a c+b^{2}}\right)^{1/4}}\right)\left(4 b-\sqrt{-4 a c+b^{2}}\right) 2^{1/4}}{4\left(-4 a c+b^{2}\right)^{3/2}\left(-b+\sqrt{-4 a c+b^{2}}\right)^{1/4}}-\frac{c^{1/4} \arctan\left(\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{-4 a c+b^{2}}\right)^{1/4}}\right)\left(4 b+\sqrt{-4 a c+b^{2}}\right) 2^{1/4}}{4\left(-4 a c+b^{2}\right)^{3/2}\left(-b-\sqrt{-4 a c+b^{2}}\right)^{1/4}}$$

$$+\frac{c^{1/4} \operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{-4 a c+b^{2}}\right)^{1/4}}\right)\left(4 b+\sqrt{-4 a c+b^{2}}\right) 2^{1/4}}{4\left(-4 a c+b^{2}\right)^{3/2}\left(-b-\sqrt{-4 a c+b^{2}}\right)^{1/4}}$$

```
Result(type 7, 120 leaves):
```

$$\frac{2\left(\frac{c x^{7/2}}{2\left(4 a c-b^{2}\right)}+\frac{b x^{3/2}}{4\left(4 a c-b^{2}\right)}\right)}{c x^{4}+b x^{2}+a}+\frac{\left(\sum_{\_R=RootOf\left(c\_Z^{8}+b\_Z^{4}+a\right)} \frac{\left(2\_R^{6} c-3\_R^{2} b\right) \ln\left(\sqrt{x}-\_R\right)}{\left(4 a c-b^{2}\right)\left(2\_R^{7} c+\_R^{3} b\right)}\right)}{8}$$

```
Problem 280: Result is not expressed in closed-form.
```

$$\int \frac{x^{3/2}}{\left(c x^{4}+b x^{2}+a\right)^{2}} dx$$

```
Optimal(type 3, 350 leaves, 9 steps):
```

$$\frac{c^{3/4}\arctan\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{-4ac+b^2}\right)^{1/4}}\right)\left(3+\frac{4b}{\sqrt{-4ac+b^2}}\right)2^{3/4}}{4\left(-4ac+b^2\right)\left(-b-\sqrt{-4ac+b^2}\right)^{3/4}}+\frac{c^{3/4}\operatorname{arctanh}\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{-4ac+b^2}\right)^{1/4}}\right)\left(3+\frac{4b}{\sqrt{-4ac+b^2}}\right)2^{3/4}}{4\left(-4ac+b^2\right)\left(-b-\sqrt{-4ac+b^2}\right)^{3/4}}$$
$$+\frac{c^{3/4}\arctan\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{-4ac+b^2}\right)^{1/4}}\right)\left(3-\frac{4b}{\sqrt{-4ac+b^2}}\right)2^{3/4}}{4\left(-4ac+b^2\right)\left(-b+\sqrt{-4ac+b^2}\right)^{3/4}}+\frac{c^{3/4}\operatorname{arctanh}\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{-4ac+b^2}\right)^{1/4}}\right)\left(3-\frac{4b}{\sqrt{-4ac+b^2}}\right)2^{3/4}}{4\left(-4ac+b^2\right)\left(-b+\sqrt{-4ac+b^2}\right)^{3/4}}$$
$$-\frac{\left(2cx^2+b\right)\sqrt{x}}{2\left(-4ac+b^2\right)\left(cx^4+bx^2+a\right)}$$

Result(type 7, 117 leaves):

$$\frac{2\left(\frac{x^{5/2}c}{2\left(4ac-b^2\right)}+\frac{\sqrt{x}\,b}{4\left(4ac-b^2\right)}\right)}{cx^4+bx^2+a}+\frac{\left(\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(6\_R^4c-b\right)\ln\left(\sqrt{x}-\_R\right)}{\left(4ac-b^2\right)\left(2\_R^7c+\_R^3b\right)}\right)}{8}$$

Problem 281: Result is not expressed in closed-form.

$$\int\frac{x^{13/2}}{\left(cx^4+bx^2+a\right)^3}\,dx$$

Optimal(type 3, 467 leaves, 10 steps):

$$\frac{x^{7/2}\left(bx^2+2a\right)}{4\left(-4ac+b^2\right)\left(cx^4+bx^2+a\right)^2}+\frac{x^{3/2}\left(24ab+\left(28ac+5b^2\right)x^2\right)}{16\left(-4ac+b^2\right)^2\left(cx^4+bx^2+a\right)}+\frac{\arctan\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{-4ac+b^2}\right)^{1/4}}\right)\left(5b^2+28ac+\frac{-172abc-5b^3}{\sqrt{-4ac+b^2}}\right)2^{1/4}}{64c^{3/4}\left(-4ac+b^2\right)^2\left(-b+\sqrt{-4ac+b^2}\right)^{1/4}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b+\sqrt{-4ac+b^2}\right)^{1/4}}\right)\left(5b^2+28ac+\frac{-172abc-5b^3}{\sqrt{-4ac+b^2}}\right)2^{1/4}}{64c^{3/4}\left(-4ac+b^2\right)^2\left(-b+\sqrt{-4ac+b^2}\right)^{1/4}}$$
$$+\frac{\arctan\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{-4ac+b^2}\right)^{1/4}}\right)\left(5b^3+172abc+\left(28ac+5b^2\right)\sqrt{-4ac+b^2}\right)2^{1/4}}{64c^{3/4}\left(-4ac+b^2\right)^{5/2}\left(-b-\sqrt{-4ac+b^2}\right)^{1/4}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{\left(-b-\sqrt{-4ac+b^2}\right)^{1/4}}\right)\left(5b^3+172abc+\left(28ac+5b^2\right)\sqrt{-4ac+b^2}\right)2^{1/4}}{64c^{3/4}\left(-4ac+b^2\right)^{5/2}\left(-b-\sqrt{-4ac+b^2}\right)^{1/4}}$$

Result(type 7, 241 leaves):

$$\frac{2\left(\frac{3a^2 b x^{3/2}}{4(16a^2c^2-8ab^2c+b^4)}-\frac{a(4ac-37b^2)x^{7/2}}{32(16a^2c^2-8ab^2c+b^4)}+\frac{9b(4ac+b^2)x^{11/2}}{32(16a^2c^2-8ab^2c+b^4)}+\frac{c(28ac+5b^2)x^{15/2}}{32(16a^2c^2-8ab^2c+b^4)}\right)}{(cx^4+bx^2+a)^2}$$
$$+\frac{\left(\sum_{\_R=RootOf(\_Z^8 c+\_Z^4 b+a)}\frac{((28ac+5b^2)\_R^6-72\_R^2 ab)\ln(\sqrt{x}-\_R)}{(16a^2c^2-8ab^2c+b^4)(2\_R^7 c+\_R^3 b)}\right)}{64}$$

Problem 282: Result is not expressed in closed-form.

$$\int\frac{x^{9/2}}{(cx^4+bx^2+a)^3}\,dx$$

Optimal(type 3, 429 leaves, 10 steps):

$$\frac{x^{3/2}(bx^2+2a)}{4(-4ac+b^2)(cx^4+bx^2+a)^2}-\frac{3x^{3/2}(8cx^2b-4ac+5b^2)}{16(-4ac+b^2)^2(cx^4+bx^2+a)}$$
$$+\frac{3c^{1/4}\arctan\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{(-b+\sqrt{-4ac+b^2})^{1/4}}\right)(11b^2+20ac-4b\sqrt{-4ac+b^2})2^{1/4}}{32(-4ac+b^2)^{5/2}(-b+\sqrt{-4ac+b^2})^{1/4}}$$
$$-\frac{3c^{1/4}\operatorname{arctanh}\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{(-b+\sqrt{-4ac+b^2})^{1/4}}\right)(11b^2+20ac-4b\sqrt{-4ac+b^2})2^{1/4}}{32(-4ac+b^2)^{5/2}(-b+\sqrt{-4ac+b^2})^{1/4}}$$
$$-\frac{3c^{1/4}\arctan\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{(-b-\sqrt{-4ac+b^2})^{1/4}}\right)(11b^2+20ac+4b\sqrt{-4ac+b^2})2^{1/4}}{32(-4ac+b^2)^{5/2}(-b-\sqrt{-4ac+b^2})^{1/4}}$$
$$+\frac{3c^{1/4}\operatorname{arctanh}\left(\frac{2^{1/4}c^{1/4}\sqrt{x}}{(-b-\sqrt{-4ac+b^2})^{1/4}}\right)(11b^2+20ac+4b\sqrt{-4ac+b^2})2^{1/4}}{32(-4ac+b^2)^{5/2}(-b-\sqrt{-4ac+b^2})^{1/4}}$$

Result(type 7, 243 leaves):

$$\frac{2\left(-\frac{a(20ac+7b^2)x^{3/2}}{32(16a^2c^2-8ab^2c+b^4)}-\frac{b(28ac+11b^2)x^{7/2}}{32(16a^2c^2-8ab^2c+b^4)}+\frac{3(4ac-13b^2)cx^{11/2}}{32(16a^2c^2-8ab^2c+b^4)}-\frac{3bc^2x^{15/2}}{4(16a^2c^2-8ab^2c+b^4)}\right)}{(cx^4+bx^2+a)^2}$$

$$+\frac{3\left(\sum_{\_R=RootOf(\_Z^8 c+\_Z^4 b+a)}\frac{\left(-8 b c\_R^6+\left(20 a c+7 b^2\right)\_R^2\right)\ln\left(\sqrt{x}-\_R\right)}{\left(16 a^2 c^2-8 a b^2 c+b^4\right)\left(2\_R^7 c+\_R^3 b\right)}\right)}{64}$$

Problem 283: Result is not expressed in closed-form.

$$\int\frac{\sqrt{x}}{\left(c x^4+b x^2+a\right)^3}\,dx$$

Optimal(type 3, 554 leaves, 10 steps):

$$\frac{x^{3/2}\left(c x^2 b-2 a c+b^2\right)}{4 a\left(-4 a c+b^2\right)\left(c x^4+b x^2+a\right)^2}+\frac{x^{3/2}\left(5 b^4-45 a b^2 c+52 a^2 c^2+b c\left(-44 a c+5 b^2\right) x^2\right)}{16 a^2\left(-4 a c+b^2\right)^2\left(c x^4+b x^2+a\right)}$$

$$-\frac{c^{1/4}\arctan\left(\frac{2^{1/4} c^{1/4}\sqrt{x}}{\left(-b-\sqrt{-4 a c+b^2}\right)^{1/4}}\right)\left(5 b^4-54 a b^2 c+520 a^2 c^2-b\left(-44 a c+5 b^2\right)\sqrt{-4 a c+b^2}\right) 2^{1/4}}{64 a^2\left(-4 a c+b^2\right)^{5/2}\left(-b-\sqrt{-4 a c+b^2}\right)^{1/4}}$$

$$+\frac{c^{1/4}\operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4}\sqrt{x}}{\left(-b-\sqrt{-4 a c+b^2}\right)^{1/4}}\right)\left(5 b^4-54 a b^2 c+520 a^2 c^2-b\left(-44 a c+5 b^2\right)\sqrt{-4 a c+b^2}\right) 2^{1/4}}{64 a^2\left(-4 a c+b^2\right)^{5/2}\left(-b-\sqrt{-4 a c+b^2}\right)^{1/4}}$$

$$+\frac{c^{1/4}\arctan\left(\frac{2^{1/4} c^{1/4}\sqrt{x}}{\left(-b+\sqrt{-4 a c+b^2}\right)^{1/4}}\right)\left(5 b^4-54 a b^2 c+520 a^2 c^2+b\left(-44 a c+5 b^2\right)\sqrt{-4 a c+b^2}\right) 2^{1/4}}{64 a^2\left(-4 a c+b^2\right)^{5/2}\left(-b+\sqrt{-4 a c+b^2}\right)^{1/4}}$$

$$-\frac{c^{1/4}\operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4}\sqrt{x}}{\left(-b+\sqrt{-4 a c+b^2}\right)^{1/4}}\right)\left(5 b^4-54 a b^2 c+520 a^2 c^2+b\left(-44 a c+5 b^2\right)\sqrt{-4 a c+b^2}\right) 2^{1/4}}{64 a^2\left(-4 a c+b^2\right)^{5/2}\left(-b+\sqrt{-4 a c+b^2}\right)^{1/4}}$$

Result(type 7, 320 leaves):

$$\frac{2\left(\frac{3\left(28 a^2 c^2-23 a b^2 c+3 b^4\right) x^{3/2}}{32 a\left(16 a^2 c^2-8 a b^2 c+b^4\right)}-\frac{b\left(8 a^2 c^2+36 a b^2 c-5 b^4\right) x^{7/2}}{32 a^2\left(16 a^2 c^2-8 a b^2 c+b^4\right)}+\frac{c\left(52 a^2 c^2-89 a b^2 c+10 b^4\right) x^{11/2}}{32 a^2\left(16 a^2 c^2-8 a b^2 c+b^4\right)}-\frac{b c^2\left(44 a c-5 b^2\right) x^{15/2}}{32 a^2\left(16 a^2 c^2-8 a b^2 c+b^4\right)}\right)}{\left(c x^4+b x^2+a\right)^2}$$

$$+\frac{\sum_{\_R=RootOf(\_Z^8 c+\_Z^4 b+a)}\frac{\left(b c\left(-44 a c+5 b^2\right)\_R^6+\left(260 a^2 c^2-49 a b^2 c+5 b^4\right)\_R^2\right)\ln\left(\sqrt{x}-\_R\right)}{\left(16 a^2 c^2-8 a b^2 c+b^4\right)\left(2\_R^7 c+\_R^3 b\right)}}{64 a^2}$$

Problem 284: Result is not expressed in closed-form.

$$\int \frac{1}{\sqrt{x}\,\left(c x^4+b x^2+a\right)^3}\, dx$$

Optimal(type 3, 554 leaves, 10 steps):

$$\frac{3 c^{3/4} \arctan\left(\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{-4 a c+b^2}\right)^{1/4}}\right) \left(7 b^4-66 a b^2 c+280 a^2 c^2-b\left(-52 a c+7 b^2\right) \sqrt{-4 a c+b^2}\right) 2^{3/4}}{64 a^2 \left(-4 a c+b^2\right)^{5/2} \left(-b-\sqrt{-4 a c+b^2}\right)^{3/4}}$$
$$+\frac{3 c^{3/4} \operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{-4 a c+b^2}\right)^{1/4}}\right) \left(7 b^4-66 a b^2 c+280 a^2 c^2-b\left(-52 a c+7 b^2\right) \sqrt{-4 a c+b^2}\right) 2^{3/4}}{64 a^2 \left(-4 a c+b^2\right)^{5/2} \left(-b-\sqrt{-4 a c+b^2}\right)^{3/4}}$$
$$-\frac{3 c^{3/4} \arctan\left(\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{-4 a c+b^2}\right)^{1/4}}\right) \left(7 b^4-66 a b^2 c+280 a^2 c^2+b\left(-52 a c+7 b^2\right) \sqrt{-4 a c+b^2}\right) 2^{3/4}}{64 a^2 \left(-4 a c+b^2\right)^{5/2} \left(-b+\sqrt{-4 a c+b^2}\right)^{3/4}}$$
$$-\frac{3 c^{3/4} \operatorname{arctanh}\left(\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{-4 a c+b^2}\right)^{1/4}}\right) \left(7 b^4-66 a b^2 c+280 a^2 c^2+b\left(-52 a c+7 b^2\right) \sqrt{-4 a c+b^2}\right) 2^{3/4}}{64 a^2 \left(-4 a c+b^2\right)^{5/2} \left(-b+\sqrt{-4 a c+b^2}\right)^{3/4}}$$
$$+\frac{\left(c x^2 b-2 a c+b^2\right) \sqrt{x}}{4 a \left(-4 a c+b^2\right) \left(c x^4+b x^2+a\right)^2}+\frac{\left(7 b^4-55 a b^2 c+60 a^2 c^2+b c\left(-52 a c+7 b^2\right) x^2\right) \sqrt{x}}{16 a^2 \left(-4 a c+b^2\right)^2 \left(c x^4+b x^2+a\right)}$$

Result(type 7, 315 leaves):

$$\frac{2\left(\frac{\left(92 a^2 c^2-79 a b^2 c+11 b^4\right) \sqrt{x}}{32\left(16 a^2 c^2-8 a b^2 c+b^4\right) a}-\frac{b\left(8 a^2 c^2+44 a b^2 c-7 b^4\right) x^{5/2}}{32 a^2\left(16 a^2 c^2-8 a b^2 c+b^4\right)}+\frac{c\left(60 a^2 c^2-107 a b^2 c+14 b^4\right) x^{9/2}}{32 a^2\left(16 a^2 c^2-8 a b^2 c+b^4\right)}-\frac{b c^2\left(52 a c-7 b^2\right) x^{13/2}}{32 a^2\left(16 a^2 c^2-8 a b^2 c+b^4\right)}\right)}{\left(c x^4+b x^2+a\right)^2}$$
$$+\frac{3\left(\sum_{\_R=RootOf(\_Z^8 c+\_Z^4 b+a)} \frac{\left(b c\left(-52 a c+7 b^2\right) \_R^4+140 a^2 c^2-59 a b^2 c+7 b^4\right) \ln\left(\sqrt{x}-\_R\right)}{\left(16 a^2 c^2-8 a b^2 c+b^4\right)\left(2 \_R^7 c+\_R^3 b\right)}\right)}{64 a^2}$$

Problem 285: Unable to integrate problem.

$$\int (d x)^{3/2} \sqrt{c x^4+b x^2+a}\, dx$$

Optimal(type 6, 121 leaves, 2 steps):

$$\frac{2\,(d\,x)^{5/2}\,\mathit{AppellF1}\left(\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2\,x^2\,c}{b-\sqrt{-4\,a\,c+b^2}},-\frac{2\,x^2\,c}{b+\sqrt{-4\,a\,c+b^2}}\right)\sqrt{c\,x^4+b\,x^2+a}}{5\,d\,\sqrt{1+\frac{2\,x^2\,c}{b-\sqrt{-4\,a\,c+b^2}}}\,\sqrt{1+\frac{2\,x^2\,c}{b+\sqrt{-4\,a\,c+b^2}}}}$$

Result(type 8, 122 leaves):

$$\frac{2\,(5\,c\,x^2+2\,b)\,x\,\sqrt{c\,x^4+b\,x^2+a}\;d^2}{45\,c\,\sqrt{d\,x}}+\frac{\left(\int-\frac{2\,(-10\,x^2\,a\,c+3\,b^2\,x^2+a\,b)}{45\,c\,\sqrt{d\,x\,(c\,x^4+b\,x^2+a)}}\,dx\right)d^2\,\sqrt{d\,x\,(c\,x^4+b\,x^2+a)}}{\sqrt{d\,x}\,\sqrt{c\,x^4+b\,x^2+a}}$$

Problem 286: Unable to integrate problem.

$$\int (d\,x)^{3/2}\,(c\,x^4+b\,x^2+a)^{3/2}\,dx$$

Optimal(type 6, 122 leaves, 2 steps):

$$\frac{2\,a\,(d\,x)^{5/2}\,\mathit{AppellF1}\left(\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2\,x^2\,c}{b-\sqrt{-4\,a\,c+b^2}},-\frac{2\,x^2\,c}{b+\sqrt{-4\,a\,c+b^2}}\right)\sqrt{c\,x^4+b\,x^2+a}}{5\,d\,\sqrt{1+\frac{2\,x^2\,c}{b-\sqrt{-4\,a\,c+b^2}}}\,\sqrt{1+\frac{2\,x^2\,c}{b+\sqrt{-4\,a\,c+b^2}}}}$$

Result(type 8, 182 leaves):

$$\frac{2\,(195\,c^3\,x^6+285\,c^2\,x^4\,b+455\,a\,c^2\,x^2+20\,c\,x^2\,b^2+176\,a\,b\,c-28\,b^3)\,x\,\sqrt{c\,x^4+b\,x^2+a}\;d^2}{3315\,c^2\,\sqrt{d\,x}}$$
$$+\frac{\left(\int-\frac{4\,(-260\,a^2\,c^2\,x^2+157\,a\,b^2\,c\,x^2-21\,b^4\,x^2+44\,a^2\,b\,c-7\,a\,b^3)}{3315\,c^2\,\sqrt{d\,x\,(c\,x^4+b\,x^2+a)}}\,dx\right)d^2\,\sqrt{d\,x\,(c\,x^4+b\,x^2+a)}}{\sqrt{d\,x}\,\sqrt{c\,x^4+b\,x^2+a}}$$

Problem 287: Unable to integrate problem.

$$\int\frac{1}{\sqrt{d\,x}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Optimal(type 6, 121 leaves, 2 steps):

$$\frac{2\,\mathit{AppellF1}\left(\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2\,x^2\,c}{b-\sqrt{-4\,a\,c+b^2}},-\frac{2\,x^2\,c}{b+\sqrt{-4\,a\,c+b^2}}\right)\sqrt{d\,x}\,\sqrt{1+\frac{2\,x^2\,c}{b-\sqrt{-4\,a\,c+b^2}}}\,\sqrt{1+\frac{2\,x^2\,c}{b+\sqrt{-4\,a\,c+b^2}}}}{d\,\sqrt{c\,x^4+b\,x^2+a}}$$

Result(type 8, 22 leaves):

$$\int \frac{1}{\sqrt{d x}\,\sqrt{c x^4 + b x^2 + a}}\, dx$$

Problem 288: Unable to integrate problem.

$$\int \frac{1}{(d x)^{3/2}\sqrt{c x^4 + b x^2 + a}}\, dx$$

Optimal(type 6, 121 leaves, 2 steps):

$$-\frac{2\, \mathit{AppellF1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{3}{4}, -\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}, -\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}\right)\sqrt{1+\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}}\,\sqrt{1+\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}}}{d\sqrt{d x}\,\sqrt{c x^4 + b x^2 + a}}$$

Result(type 8, 100 leaves):

$$-\frac{2\sqrt{c x^4 + b x^2 + a}}{a d\sqrt{d x}} + \frac{\left(\int \frac{x\left(3 c x^2 + b\right)}{a\sqrt{d x\left(c x^4 + b x^2 + a\right)}}\, dx\right)\sqrt{d x\left(c x^4 + b x^2 + a\right)}}{d\sqrt{d x}\,\sqrt{c x^4 + b x^2 + a}}$$

Problem 289: Unable to integrate problem.

$$\int \frac{(d x)^{3/2}}{\left(c x^4 + b x^2 + a\right)^{3/2}}\, dx$$

Optimal(type 6, 124 leaves, 2 steps):

$$\frac{2\,(d x)^{5/2}\, \mathit{AppellF1}\left(\frac{5}{4}, \frac{3}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}, -\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}\right)\sqrt{1+\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}}\,\sqrt{1+\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}}}{5 a d\sqrt{c x^4 + b x^2 + a}}$$

Result(type 8, 22 leaves):

$$\int \frac{(d x)^{3/2}}{\left(c x^4 + b x^2 + a\right)^{3/2}}\, dx$$

Problem 290: Unable to integrate problem.

$$\int \frac{1}{(d x)^{3/2}\left(c x^4 + b x^2 + a\right)^{3/2}}\, dx$$

Optimal(type 6, 124 leaves, 2 steps):

$$-\frac{2\,\mathrm{AppellF1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{3}{4},-\frac{2x^2c}{b-\sqrt{-4ac+b^2}},-\frac{2x^2c}{b+\sqrt{-4ac+b^2}}\right)\sqrt{1+\frac{2x^2c}{b-\sqrt{-4ac+b^2}}}\,\sqrt{1+\frac{2x^2c}{b+\sqrt{-4ac+b^2}}}}{a\,d\sqrt{dx}\,\sqrt{cx^4+bx^2+a}}$$

Result(type 8, 144 leaves):

$$-\frac{2\sqrt{cx^4+bx^2+a}}{a^2d\sqrt{dx}}+\frac{\left(\int\frac{x\left(3c^2x^6+4bcx^4+2x^2ac+b^2x^2\right)}{a^2c\left(x^4+\frac{bx^2}{c}+\frac{a}{c}\right)\sqrt{dx\left(cx^4+bx^2+a\right)}}\,dx\right)\sqrt{dx\left(cx^4+bx^2+a\right)}}{d\sqrt{dx}\,\sqrt{cx^4+bx^2+a}}$$

Problem 292: Unable to integrate problem.

$$\int(dx)^m\left(cx^4+bx^2+a\right)^{3/2}dx$$

Optimal(type 6, 136 leaves, 2 steps):

$$\frac{a\,(dx)^{1+m}\,\mathrm{AppellF1}\left(\frac{1}{2}+\frac{m}{2},-\frac{3}{2},-\frac{3}{2},\frac{3}{2}+\frac{m}{2},-\frac{2x^2c}{b-\sqrt{-4ac+b^2}},-\frac{2x^2c}{b+\sqrt{-4ac+b^2}}\right)\sqrt{cx^4+bx^2+a}}{d\,(1+m)\sqrt{1+\frac{2x^2c}{b-\sqrt{-4ac+b^2}}}\,\sqrt{1+\frac{2x^2c}{b+\sqrt{-4ac+b^2}}}}$$

Result(type 8, 22 leaves):

$$\int(dx)^m\left(cx^4+bx^2+a\right)^{3/2}dx$$

Problem 293: Unable to integrate problem.

$$\int\frac{(dx)^m}{\left(cx^4+bx^2+a\right)^{3/2}}\,dx$$

Optimal(type 6, 138 leaves, 2 steps):

$$\frac{(dx)^{1+m}\,\mathrm{AppellF1}\left(\frac{1}{2}+\frac{m}{2},\frac{3}{2},\frac{3}{2},\frac{3}{2}+\frac{m}{2},-\frac{2x^2c}{b-\sqrt{-4ac+b^2}},-\frac{2x^2c}{b+\sqrt{-4ac+b^2}}\right)\sqrt{1+\frac{2x^2c}{b-\sqrt{-4ac+b^2}}}\,\sqrt{1+\frac{2x^2c}{b+\sqrt{-4ac+b^2}}}}{a\,d\,(1+m)\sqrt{cx^4+bx^2+a}}$$

Result(type 8, 22 leaves):

$$\int\frac{(dx)^m}{\left(cx^4+bx^2+a\right)^{3/2}}\,dx$$

Problem 294: Unable to integrate problem.

$$\int x^5 \left(c x^4+b x^2+a\right)^p \mathrm{d}x$$

Optimal(type 5, 208 leaves, 4 steps):

$$-\frac{b (2+p) \left(c x^4+b x^2+a\right)^{1+p}}{4 c^2 (1+p) (3+2 p)}+\frac{x^2 \left(c x^4+b x^2+a\right)^{1+p}}{2 c (3+2 p)}$$
$$+\frac{2^{-1+p} \left(2 a c-b^2 (2+p)\right) \left(c x^4+b x^2+a\right)^{1+p} \mathrm{hypergeom}\left([-p,1+p],[2+p],\frac{2 c x^2+\sqrt{-4 a c+b^2}+b}{2 \sqrt{-4 a c+b^2}}\right) \left(\frac{-2 c x^2+\sqrt{-4 a c+b^2}-b}{\sqrt{-4 a c+b^2}}\right)^{-1-p}}{c^2 (1+p) (3+2 p) \sqrt{-4 a c+b^2}}$$

Result(type 8, 20 leaves):

$$\int x^5 \left(c x^4+b x^2+a\right)^p \mathrm{d}x$$

Problem 295: Unable to integrate problem.

$$\int x^3 \left(c x^4+b x^2+a\right)^p \mathrm{d}x$$

Optimal(type 5, 147 leaves, 3 steps):

$$\frac{\left(c x^4+b x^2+a\right)^{1+p}}{4 c (1+p)}+\frac{2^{-1+p} b \left(c x^4+b x^2+a\right)^{1+p} \mathrm{hypergeom}\left([-p,1+p],[2+p],\frac{2 c x^2+\sqrt{-4 a c+b^2}+b}{2 \sqrt{-4 a c+b^2}}\right) \left(\frac{-2 c x^2+\sqrt{-4 a c+b^2}-b}{\sqrt{-4 a c+b^2}}\right)^{-1-p}}{c (1+p) \sqrt{-4 a c+b^2}}$$

Result(type 8, 20 leaves):

$$\int x^3 \left(c x^4+b x^2+a\right)^p \mathrm{d}x$$

Problem 296: Unable to integrate problem.

$$\int \frac{\left(c x^4+b x^2+a\right)^p}{x^3} \mathrm{d}x$$

Optimal(type 6, 156 leaves, 3 steps):

$$-\frac{2^{-1+2 p} \left(c x^4+b x^2+a\right)^p \mathit{AppellF1}\left(1-2 p,-p,-p,2-2 p,\frac{-b-\sqrt{-4 a c+b^2}}{2 c x^2},\frac{-b+\sqrt{-4 a c+b^2}}{2 c x^2}\right)}{(1-2 p) x^2 \left(\frac{2 c x^2-\sqrt{-4 a c+b^2}+b}{c x^2}\right)^p \left(\frac{2 c x^2+\sqrt{-4 a c+b^2}+b}{c x^2}\right)^p}$$

Result(type 8, 20 leaves):

$$\int \frac{(c x^4 + b x^2 + a)^p}{x^3}\, dx$$

Problem 297: Unable to integrate problem.

$$\int \frac{(c x^4 + b x^2 + a)^p}{x^5}\, dx$$

Optimal(type 6, 154 leaves, 3 steps):

$$-\frac{4^{-1+p} (c x^4 + b x^2 + a)^p \, AppellF1\left(2 - 2 p, -p, -p, 3 - 2 p, \frac{-b - \sqrt{-4 a c + b^2}}{2 c x^2}, \frac{-b + \sqrt{-4 a c + b^2}}{2 c x^2}\right)}{(1 - p)\, x^4 \left(\frac{2 c x^2 - \sqrt{-4 a c + b^2} + b}{c x^2}\right)^p \left(\frac{2 c x^2 + \sqrt{-4 a c + b^2} + b}{c x^2}\right)^p}$$

Result(type 8, 20 leaves):

$$\int \frac{(c x^4 + b x^2 + a)^p}{x^5}\, dx$$

Test results for the 109 problems in "1.2.2.3 (d+e x^2)^m (a+b x^2+c x^4)^p.txt"

Problem 2: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2 d + c}{-b x^4 + a}\, dx$$

Optimal(type 3, 58 leaves, 3 steps):

$$\frac{\arctan\left(\frac{b^{1/4} x}{a^{1/4}}\right) \left(-d \sqrt{a} + c \sqrt{b}\right)}{2 a^{3/4} b^{3/4}} + \frac{\operatorname{arctanh}\left(\frac{b^{1/4} x}{a^{1/4}}\right) \left(d \sqrt{a} + c \sqrt{b}\right)}{2 a^{3/4} b^{3/4}}$$

Result(type 3, 121 leaves):

$$\frac{c \left(\frac{a}{b}\right)^{1/4} \ln\left(\frac{x + \left(\frac{a}{b}\right)^{1/4}}{x - \left(\frac{a}{b}\right)^{1/4}}\right)}{4 a} + \frac{c \left(\frac{a}{b}\right)^{1/4} \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{1/4}}\right)}{2 a} - \frac{d \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{1/4}}\right)}{2 b \left(\frac{a}{b}\right)^{1/4}} + \frac{d \ln\left(\frac{x + \left(\frac{a}{b}\right)^{1/4}}{x - \left(\frac{a}{b}\right)^{1/4}}\right)}{4 b \left(\frac{a}{b}\right)^{1/4}}$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \frac{-b x^2+\sqrt{a}\,\sqrt{b}}{b x^4+a}\, dx$$

Optimal(type 3, 70 leaves, 3 steps):

$$-\frac{b^{1/4}\ln\left(-a^{1/4} b^{1/4} x\sqrt{2}+\sqrt{a}+x^2\sqrt{b}\right)\sqrt{2}}{4 a^{1/4}}+\frac{b^{1/4}\ln\left(a^{1/4} b^{1/4} x\sqrt{2}+\sqrt{a}+x^2\sqrt{b}\right)\sqrt{2}}{4 a^{1/4}}$$

Result(type 3, 253 leaves):

$$\frac{\sqrt{b}\left(\frac{a}{b}\right)^{1/4}\sqrt{2}\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{1/4} x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{1/4} x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{8\sqrt{a}}+\frac{\sqrt{b}\left(\frac{a}{b}\right)^{1/4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{1/4}}+1\right)}{4\sqrt{a}}+\frac{\sqrt{b}\left(\frac{a}{b}\right)^{1/4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{1/4}}-1\right)}{4\sqrt{a}}$$

$$-\frac{\sqrt{2}\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{1/4} x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{1/4} x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{8\left(\frac{a}{b}\right)^{1/4}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{1/4}}+1\right)}{4\left(\frac{a}{b}\right)^{1/4}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{1/4}}-1\right)}{4\left(\frac{a}{b}\right)^{1/4}}$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \frac{-b x^2+1}{\sqrt{-b^2 x^4+1}}\, dx$$

Optimal(type 4, 27 leaves, 5 steps):

$$-\frac{\mathrm{EllipticE}\left(x\sqrt{b}, \mathrm{I}\right)}{\sqrt{b}}+\frac{2\,\mathrm{EllipticF}\left(x\sqrt{b}, \mathrm{I}\right)}{\sqrt{b}}$$

Result(type 4, 98 leaves):

$$\frac{\sqrt{-b x^2+1}\,\sqrt{b x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{b}, \mathrm{I}\right)-\mathrm{EllipticE}\left(x\sqrt{b}, \mathrm{I}\right)\right)}{\sqrt{b}\,\sqrt{-b^2 x^4+1}}+\frac{\sqrt{-b x^2+1}\,\sqrt{b x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{b}, \mathrm{I}\right)}{\sqrt{b}\,\sqrt{-b^2 x^4+1}}$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int \frac{b x^2+1}{\sqrt{b^2 x^4-1}}\, dx$$

Optimal(type 4, 35 leaves, 3 steps):

$$\frac{\operatorname{EllipticE}\left(x\sqrt{b}, \mathrm{I}\right)\sqrt{-b^2 x^4+1}}{\sqrt{b}\,\sqrt{b^2 x^4-1}}$$

Result(type 4, 106 leaves):

$$\frac{\sqrt{b x^2+1}\,\sqrt{-b x^2+1}\,\operatorname{EllipticF}\left(\sqrt{-b}\,x, \mathrm{I}\right)}{\sqrt{-b}\,\sqrt{b^2 x^4-1}}+\frac{\sqrt{b x^2+1}\,\sqrt{-b x^2+1}\,\left(\operatorname{EllipticF}\left(\sqrt{-b}\,x, \mathrm{I}\right)-\operatorname{EllipticE}\left(\sqrt{-b}\,x, \mathrm{I}\right)\right)}{\sqrt{-b}\,\sqrt{b^2 x^4-1}}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \frac{c^2 x^2+1}{\sqrt{-c^4 x^4+1}}\,\mathrm{d}x$$

Optimal(type 4, 10 leaves, 2 steps):

$$\frac{\operatorname{EllipticE}(c x, \mathrm{I})}{c}$$

Result(type 4, 117 leaves):

$$\frac{\sqrt{-c^2 x^2+1}\,\sqrt{c^2 x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{c^2}, \mathrm{I}\right)}{\sqrt{c^2}\,\sqrt{-c^4 x^4+1}}-\frac{\sqrt{-c^2 x^2+1}\,\sqrt{c^2 x^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{c^2}, \mathrm{I}\right)-\operatorname{EllipticE}\left(x\sqrt{c^2}, \mathrm{I}\right)\right)}{\sqrt{c^2}\,\sqrt{-c^4 x^4+1}}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{e x^2+d}{\frac{c d^2}{e^2}+b x^2+c x^4}\,\mathrm{d}x$$

Optimal(type 3, 102 leaves, 5 steps):

$$-\frac{e^{3/2}\arctan\left(\frac{-2 x\sqrt{c}\,\sqrt{e}+\sqrt{-b e+2 c d}}{\sqrt{b e+2 c d}}\right)}{\sqrt{c}\,\sqrt{b e+2 c d}}+\frac{e^{3/2}\arctan\left(\frac{2 x\sqrt{c}\,\sqrt{e}+\sqrt{-b e+2 c d}}{\sqrt{b e+2 c d}}\right)}{\sqrt{c}\,\sqrt{b e+2 c d}}$$

Result(type 3, 581 leaves):

$$\frac{e^4\sqrt{2}\,\operatorname{arctanh}\left(\frac{c e x\sqrt{2}}{\sqrt{\left(-e^2 b+\sqrt{e^2 (b e-2 c d)(b e+2 c d)}\right) c}}\right) b}{2\sqrt{e^2 (b e-2 c d)(b e+2 c d)}\,\sqrt{\left(-e^2 b+\sqrt{e^2 (b e-2 c d)(b e+2 c d)}\right) c}}$$

$$- \frac{e^3 c \sqrt{2}\, \arctan\!\left(\frac{c e x \sqrt{2}}{\sqrt{\left(-e^2 b + \sqrt{e^2 (b e - 2 c d) (b e + 2 c d)}\right) c}}\right) d}{\sqrt{e^2 (b e - 2 c d) (b e + 2 c d)}\, \sqrt{\left(-e^2 b + \sqrt{e^2 (b e - 2 c d) (b e + 2 c d)}\right) c}} - \frac{e^2 \sqrt{2}\, \operatorname{arctanh}\!\left(\frac{c e x \sqrt{2}}{\sqrt{\left(-e^2 b + \sqrt{e^2 (b e - 2 c d) (b e + 2 c d)}\right) c}}\right)}{2 \sqrt{\left(-e^2 b + \sqrt{e^2 (b e - 2 c d) (b e + 2 c d)}\right) c}}$$

$$+ \frac{e^4 \sqrt{2}\, \arctan\!\left(\frac{c e x \sqrt{2}}{\sqrt{\left(e^2 b + \sqrt{e^2 (b e - 2 c d) (b e + 2 c d)}\right) c}}\right) b}{2 \sqrt{e^2 (b e - 2 c d) (b e + 2 c d)}\, \sqrt{\left(e^2 b + \sqrt{e^2 (b e - 2 c d) (b e + 2 c d)}\right) c}}$$

$$- \frac{e^3 c \sqrt{2}\, \arctan\!\left(\frac{c e x \sqrt{2}}{\sqrt{\left(e^2 b + \sqrt{e^2 (b e - 2 c d) (b e + 2 c d)}\right) c}}\right) d}{\sqrt{e^2 (b e - 2 c d) (b e + 2 c d)}\, \sqrt{\left(e^2 b + \sqrt{e^2 (b e - 2 c d) (b e + 2 c d)}\right) c}} + \frac{e^2 \sqrt{2}\, \arctan\!\left(\frac{c e x \sqrt{2}}{\sqrt{\left(e^2 b + \sqrt{e^2 (b e - 2 c d) (b e + 2 c d)}\right) c}}\right)}{2 \sqrt{\left(e^2 b + \sqrt{e^2 (b e - 2 c d) (b e + 2 c d)}\right) c}}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \frac{2 x^2 + 1}{4 x^4 + b x^2 + 1}\, dx$$

Optimal(type 3, 50 leaves, 5 steps):

$$-\frac{\arctan\!\left(\frac{-4 x + \sqrt{4 - b}}{\sqrt{4 + b}}\right)}{\sqrt{4 + b}} + \frac{\arctan\!\left(\frac{4 x + \sqrt{4 - b}}{\sqrt{4 + b}}\right)}{\sqrt{4 + b}}$$

Result(type 3, 276 leaves):

$$\frac{4 \arctan\!\left(\frac{4 x}{\sqrt{-2 \sqrt{(-4 + b) (4 + b)} + 2 b}}\right)}{\sqrt{(-4 + b) (4 + b)}\, \sqrt{-2 \sqrt{(-4 + b) (4 + b)} + 2 b}} + \frac{\arctan\!\left(\frac{4 x}{\sqrt{-2 \sqrt{(-4 + b) (4 + b)} + 2 b}}\right)}{\sqrt{-2 \sqrt{(-4 + b) (4 + b)} + 2 b}}$$

$$- \frac{\arctan\!\left(\frac{4 x}{\sqrt{-2 \sqrt{(-4 + b) (4 + b)} + 2 b}}\right) b}{\sqrt{(-4 + b) (4 + b)}\, \sqrt{-2 \sqrt{(-4 + b) (4 + b)} + 2 b}} - \frac{4 \arctan\!\left(\frac{4 x}{\sqrt{2 \sqrt{(-4 + b) (4 + b)} + 2 b}}\right)}{\sqrt{(-4 + b) (4 + b)}\, \sqrt{2 \sqrt{(-4 + b) (4 + b)} + 2 b}}$$

$$+ \frac{\arctan\!\left(\frac{4 x}{\sqrt{2 \sqrt{(-4 + b) (4 + b)} + 2 b}}\right)}{\sqrt{2 \sqrt{(-4 + b) (4 + b)} + 2 b}} + \frac{\arctan\!\left(\frac{4 x}{\sqrt{2 \sqrt{(-4 + b) (4 + b)} + 2 b}}\right) b}{\sqrt{(-4 + b) (4 + b)}\, \sqrt{2 \sqrt{(-4 + b) (4 + b)} + 2 b}}$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int \frac{2x^2+1}{4x^4+6x^2+1}\, dx$$

Optimal(type 3, 45 leaves, 3 steps):

$$\frac{\arctan\left(\frac{2x}{\frac{\sqrt{10}}{2}-\frac{\sqrt{2}}{2}}\right)\sqrt{10}}{10}+\frac{\arctan\left(\frac{2x}{\frac{\sqrt{10}}{2}+\frac{\sqrt{2}}{2}}\right)\sqrt{10}}{10}$$

Result(type 3, 135 leaves):

$$\frac{2\sqrt{5}\arctan\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}+\frac{2\arctan\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{2\sqrt{10}+2\sqrt{2}}-\frac{2\sqrt{5}\arctan\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)}+\frac{2\arctan\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{2\sqrt{10}-2\sqrt{2}}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2+1}{x^4+4x^2+1}\, dx$$

Optimal(type 3, 43 leaves, 3 steps):

$$\frac{\arctan\left(\frac{x}{\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}}\right)\sqrt{6}}{6}+\frac{\arctan\left(\frac{x}{\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}}\right)\sqrt{6}}{6}$$

Result(type 3, 109 leaves):

$$\frac{\sqrt{3}\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}+\frac{\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{\sqrt{6}+\sqrt{2}}-\frac{\sqrt{3}\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\left(\sqrt{6}-\sqrt{2}\right)}+\frac{\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{\sqrt{6}-\sqrt{2}}$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \frac{-x^2+1}{x^4+b x^2+1}\, dx$$

Optimal(type 3, 50 leaves, 3 steps):

$$-\frac{\ln\left(1+x^2-x\sqrt{2-b}\right)}{2\sqrt{2-b}}+\frac{\ln\left(1+x^2+x\sqrt{2-b}\right)}{2\sqrt{2-b}}$$

Result(type 3, 278 leaves):

$$
-\frac{2\arctan\left(\frac{2x}{\sqrt{2\sqrt{(-2+b)(b+2)}+2b}}\right)}{\sqrt{(-2+b)(b+2)}\sqrt{2\sqrt{(-2+b)(b+2)}+2b}}-\frac{\arctan\left(\frac{2x}{\sqrt{2\sqrt{(-2+b)(b+2)}+2b}}\right)}{\sqrt{2\sqrt{(-2+b)(b+2)}+2b}}-\frac{\arctan\left(\frac{2x}{\sqrt{2\sqrt{(-2+b)(b+2)}+2b}}\right)b}{\sqrt{(-2+b)(b+2)}\sqrt{2\sqrt{(-2+b)(b+2)}+2b}}
$$

$$
+\frac{2\arctan\left(\frac{2x}{\sqrt{-2\sqrt{(-2+b)(b+2)}+2b}}\right)}{\sqrt{(-2+b)(b+2)}\sqrt{-2\sqrt{(-2+b)(b+2)}+2b}}-\frac{\arctan\left(\frac{2x}{\sqrt{-2\sqrt{(-2+b)(b+2)}+2b}}\right)}{\sqrt{-2\sqrt{(-2+b)(b+2)}+2b}}
$$

$$
+\frac{\arctan\left(\frac{2x}{\sqrt{-2\sqrt{(-2+b)(b+2)}+2b}}\right)b}{\sqrt{(-2+b)(b+2)}\sqrt{-2\sqrt{(-2+b)(b+2)}+2b}}
$$

Problem 25: Result more than twice size of optimal antiderivative.

$$
\int \frac{-x^2+1}{x^4+5x^2+1}\,dx
$$

Optimal(type 3, 38 leaves, 3 steps):

$$
-\frac{\arctan\left(\frac{x\sqrt{2}}{\sqrt{5+\sqrt{21}}}\right)\sqrt{3}}{3}+\frac{\arctan\left(x\left(\frac{\sqrt{7}}{2}+\frac{\sqrt{3}}{2}\right)\right)\sqrt{3}}{3}
$$

Result(type 3, 135 leaves):

$$
-\frac{2\sqrt{21}\arctan\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{3\left(2\sqrt{7}+2\sqrt{3}\right)}-\frac{2\arctan\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{2\sqrt{7}+2\sqrt{3}}+\frac{2\sqrt{21}\arctan\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{3\left(2\sqrt{7}-2\sqrt{3}\right)}-\frac{2\arctan\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{2\sqrt{7}-2\sqrt{3}}
$$

Problem 26: Result more than twice size of optimal antiderivative.

$$
\int \frac{-x^2+1}{x^4-5x^2+1}\,dx
$$

Optimal(type 3, 37 leaves, 5 steps):

$$
-\frac{\operatorname{arctanh}\left(\frac{\left(-2x+\sqrt{3}\right)\sqrt{7}}{7}\right)\sqrt{7}}{7}+\frac{\operatorname{arctanh}\left(\frac{\left(2x+\sqrt{3}\right)\sqrt{7}}{7}\right)\sqrt{7}}{7}
$$

Result(type 3, 81 leaves):

$$
\frac{2\left(3+\sqrt{21}\right)\sqrt{21}\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21\left(2\sqrt{7}+2\sqrt{3}\right)}+\frac{2\left(-3+\sqrt{21}\right)\sqrt{21}\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21\left(2\sqrt{7}-2\sqrt{3}\right)}
$$

Problem 28: Result more than twice size of optimal antiderivative.

$$\int \frac{b x^2 + a}{x^4 + x^2 + 2}\, dx$$

Optimal(type 3, 162 leaves, 9 steps):

$$-\frac{\arctan\left(\frac{-2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)\left(a+b\sqrt{2}\right)\sqrt{-14+28\sqrt{2}}}{28}+\frac{\arctan\left(\frac{2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)\left(a+b\sqrt{2}\right)\sqrt{-14+28\sqrt{2}}}{28}$$
$$-\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{-1+2\sqrt{2}}\right)\left(a-b\sqrt{2}\right)}{4\sqrt{-2+4\sqrt{2}}}+\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)\left(a-b\sqrt{2}\right)}{4\sqrt{-2+4\sqrt{2}}}$$

Result(type 3, 709 leaves):

$$\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,a}{56}-\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,b}{14}$$
$$+\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)\sqrt{-1+2\sqrt{2}}\,a}{14}-\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)\sqrt{-1+2\sqrt{2}}\,b}{28}$$
$$-\frac{\arctan\left(\frac{2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)\left(-1+2\sqrt{2}\right)\sqrt{2}\,a}{28\sqrt{1+2\sqrt{2}}}+\frac{\arctan\left(\frac{2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)\left(-1+2\sqrt{2}\right)\sqrt{2}\,b}{7\sqrt{1+2\sqrt{2}}}$$
$$-\frac{\arctan\left(\frac{2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)\left(-1+2\sqrt{2}\right)a}{7\sqrt{1+2\sqrt{2}}}+\frac{\arctan\left(\frac{2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)\left(-1+2\sqrt{2}\right)b}{14\sqrt{1+2\sqrt{2}}}+\frac{\arctan\left(\frac{2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)\sqrt{2}\,a}{2\sqrt{1+2\sqrt{2}}}$$
$$-\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{-1+2\sqrt{2}}\right)\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,a}{56}+\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{-1+2\sqrt{2}}\right)\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,b}{14}$$
$$-\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{-1+2\sqrt{2}}\right)\sqrt{-1+2\sqrt{2}}\,a}{14}+\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{-1+2\sqrt{2}}\right)\sqrt{-1+2\sqrt{2}}\,b}{28}$$
$$-\frac{\arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)\left(-1+2\sqrt{2}\right)\sqrt{2}\,a}{28\sqrt{1+2\sqrt{2}}}+\frac{\arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)\left(-1+2\sqrt{2}\right)\sqrt{2}\,b}{7\sqrt{1+2\sqrt{2}}}$$

$$-\frac{\arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)\left(-1+2\sqrt{2}\right)a}{7\sqrt{1+2\sqrt{2}}}+\frac{\arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)\left(-1+2\sqrt{2}\right)b}{14\sqrt{1+2\sqrt{2}}}+\frac{\arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)\sqrt{2}\,a}{2\sqrt{1+2\sqrt{2}}}$$

Problem 29: Result more than twice size of optimal antiderivative.

$$\int \frac{b x^2+a}{\left(x^4+x^2+2\right)^2}\,dx$$

Optimal(type 3, 235 leaves, 10 steps):

$$\frac{x\left(3a+2b-(a-4b)x^2\right)}{28\left(x^4+x^2+2\right)}-\frac{\arctan\left(\frac{-2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)\left(-b\left(2-4\sqrt{2}\right)+a\left(11-\sqrt{2}\right)\right)\sqrt{-14+28\sqrt{2}}}{784}$$
$$+\frac{\arctan\left(\frac{2x+\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)\left(-b\left(2-4\sqrt{2}\right)+a\left(11-\sqrt{2}\right)\right)\sqrt{-14+28\sqrt{2}}}{784}-\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{-1+2\sqrt{2}}\right)\left(11a-2b+(a-4b)\sqrt{2}\right)}{112\sqrt{-2+4\sqrt{2}}}$$
$$+\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)\left(a\left(11+\sqrt{2}\right)-2b-4b\sqrt{2}\right)}{112\sqrt{-2+4\sqrt{2}}}$$

Result(type 3, 1505 leaves):

$$\frac{53\ln\left(\left(1+2\sqrt{2}\right)\left(x^2+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)\right)\sqrt{-1+2\sqrt{2}}\,a}{784\left(1+2\sqrt{2}\right)}-\frac{11\ln\left(\left(1+2\sqrt{2}\right)\left(x^2+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)\right)\sqrt{-1+2\sqrt{2}}\,b}{196\left(1+2\sqrt{2}\right)}$$
$$+\frac{11\arctan\left(\frac{2\left(1+2\sqrt{2}\right)x+\sqrt{-1+2\sqrt{2}}\left(1+2\sqrt{2}\right)}{\sqrt{22\sqrt{2}+25}}\right)\sqrt{2}\,a}{56\sqrt{22\sqrt{2}+25}}-\frac{\arctan\left(\frac{2\left(1+2\sqrt{2}\right)x+\sqrt{-1+2\sqrt{2}}\left(1+2\sqrt{2}\right)}{\sqrt{22\sqrt{2}+25}}\right)\sqrt{2}\,b}{28\sqrt{22\sqrt{2}+25}}$$
$$-\frac{53\arctan\left(\frac{2\left(1+2\sqrt{2}\right)x+\sqrt{-1+2\sqrt{2}}\left(1+2\sqrt{2}\right)}{\sqrt{22\sqrt{2}+25}}\right)\left(-1+2\sqrt{2}\right)a}{392\sqrt{22\sqrt{2}+25}}$$

$$+\frac{11\arctan\left(\frac{2\left(1+2\sqrt{2}\right)x+\sqrt{-1+2\sqrt{2}}\left(1+2\sqrt{2}\right)}{\sqrt{22\sqrt{2}+25}}\right)\left(-1+2\sqrt{2}\right)b}{98\sqrt{22\sqrt{2}+25}}-\frac{53\ln\left(-\left(1+2\sqrt{2}\right)\left(x\sqrt{-1+2\sqrt{2}}-x^2-\sqrt{2}\right)\right)\sqrt{-1+2\sqrt{2}}\,a}{784\left(1+2\sqrt{2}\right)}$$

$$+\frac{11\ln\left(-\left(1+2\sqrt{2}\right)\left(x\sqrt{-1+2\sqrt{2}}-x^2-\sqrt{2}\right)\right)\sqrt{-1+2\sqrt{2}}\,b}{196\left(1+2\sqrt{2}\right)}+\frac{11\arctan\left(\frac{2\left(1+2\sqrt{2}\right)x-\sqrt{-1+2\sqrt{2}}\left(1+2\sqrt{2}\right)}{\sqrt{22\sqrt{2}+25}}\right)\sqrt{2}\,a}{56\sqrt{22\sqrt{2}+25}}$$

$$-\frac{\arctan\left(\frac{2\left(1+2\sqrt{2}\right)x-\sqrt{-1+2\sqrt{2}}\left(1+2\sqrt{2}\right)}{\sqrt{22\sqrt{2}+25}}\right)\sqrt{2}\,b}{28\sqrt{22\sqrt{2}+25}}-\frac{53\arctan\left(\frac{2\left(1+2\sqrt{2}\right)x-\sqrt{-1+2\sqrt{2}}\left(1+2\sqrt{2}\right)}{\sqrt{22\sqrt{2}+25}}\right)\left(-1+2\sqrt{2}\right)a}{392\sqrt{22\sqrt{2}+25}}$$

$$+\frac{11\arctan\left(\frac{2\left(1+2\sqrt{2}\right)x-\sqrt{-1+2\sqrt{2}}\left(1+2\sqrt{2}\right)}{\sqrt{22\sqrt{2}+25}}\right)\left(-1+2\sqrt{2}\right)b}{98\sqrt{22\sqrt{2}+25}}$$

$$+\frac{107\ln\left(\left(1+2\sqrt{2}\right)\left(x^2+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)\right)\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,a}{1568\left(1+2\sqrt{2}\right)}-\frac{25\ln\left(\left(1+2\sqrt{2}\right)\left(x^2+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)\right)\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,b}{784\left(1+2\sqrt{2}\right)}$$

$$-\frac{107\arctan\left(\frac{2\left(1+2\sqrt{2}\right)x+\sqrt{-1+2\sqrt{2}}\left(1+2\sqrt{2}\right)}{\sqrt{22\sqrt{2}+25}}\right)\left(-1+2\sqrt{2}\right)\sqrt{2}\,a}{784\sqrt{22\sqrt{2}+25}}$$

$$+\frac{25\arctan\left(\frac{2\left(1+2\sqrt{2}\right)x+\sqrt{-1+2\sqrt{2}}\left(1+2\sqrt{2}\right)}{\sqrt{22\sqrt{2}+25}}\right)\left(-1+2\sqrt{2}\right)\sqrt{2}\,b}{392\sqrt{22\sqrt{2}+25}}$$

$$-\frac{107\ln\left(-\left(1+2\sqrt{2}\right)\left(x\sqrt{-1+2\sqrt{2}}-x^2-\sqrt{2}\right)\right)\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,a}{1568\left(1+2\sqrt{2}\right)}$$

$$+\frac{25\ln\left(-\left(1+2\sqrt{2}\right)\left(x\sqrt{-1+2\sqrt{2}}-x^2-\sqrt{2}\right)\right)\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,b}{784\left(1+2\sqrt{2}\right)}$$

$$- \frac{107 \arctan\left(\frac{2\left(1+2\sqrt{2}\right)x-\sqrt{-1+2\sqrt{2}}\,\left(1+2\sqrt{2}\right)}{\sqrt{22\sqrt{2}+25}}\right)\left(-1+2\sqrt{2}\right)\sqrt{2}\,a}{784\sqrt{22\sqrt{2}+25}}$$

$$+ \frac{25 \arctan\left(\frac{2\left(1+2\sqrt{2}\right)x-\sqrt{-1+2\sqrt{2}}\,\left(1+2\sqrt{2}\right)}{\sqrt{22\sqrt{2}+25}}\right)\left(-1+2\sqrt{2}\right)\sqrt{2}\,b}{392\sqrt{22\sqrt{2}+25}} + \frac{11 \arctan\left(\frac{2\left(1+2\sqrt{2}\right)x+\sqrt{-1+2\sqrt{2}}\,\left(1+2\sqrt{2}\right)}{\sqrt{22\sqrt{2}+25}}\right)a}{14\sqrt{22\sqrt{2}+25}}$$

$$- \frac{\arctan\left(\frac{2\left(1+2\sqrt{2}\right)x+\sqrt{-1+2\sqrt{2}}\,\left(1+2\sqrt{2}\right)}{\sqrt{22\sqrt{2}+25}}\right)b}{7\sqrt{22\sqrt{2}+25}} + \frac{11 \arctan\left(\frac{2\left(1+2\sqrt{2}\right)x-\sqrt{-1+2\sqrt{2}}\,\left(1+2\sqrt{2}\right)}{\sqrt{22\sqrt{2}+25}}\right)a}{14\sqrt{22\sqrt{2}+25}}$$

$$- \frac{\arctan\left(\frac{2\left(1+2\sqrt{2}\right)x-\sqrt{-1+2\sqrt{2}}\,\left(1+2\sqrt{2}\right)}{\sqrt{22\sqrt{2}+25}}\right)b}{7\sqrt{22\sqrt{2}+25}}$$

$$+ \frac{\frac{\left(-14a-28\sqrt{2}\,a+112b\sqrt{2}+56b\right)x}{1+2\sqrt{2}} + \frac{\sqrt{-1+2\sqrt{2}}\,\left(-70a-42\sqrt{2}\,a+56b\sqrt{2}+28b\right)}{1+2\sqrt{2}}}{784\left(x^2+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)}$$

$$- \frac{-\frac{\left(-14a-28\sqrt{2}\,a+112b\sqrt{2}+56b\right)x}{1+2\sqrt{2}} + \frac{\sqrt{-1+2\sqrt{2}}\,\left(-70a-42\sqrt{2}\,a+56b\sqrt{2}+28b\right)}{1+2\sqrt{2}}}{784\left(x^2+\sqrt{2}-x\sqrt{-1+2\sqrt{2}}\right)}$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int \frac{-x^2+3}{\sqrt{-x^4+x^2+3}}\,dx$$

Optimal(type 4, 74 leaves, 4 steps):

$$-\frac{\mathrm{EllipticE}\left(\frac{x\sqrt{2}}{\sqrt{1+\sqrt{13}}}, \frac{I\sqrt{3}}{6}+\frac{I\sqrt{39}}{6}\right)\sqrt{-2+2\sqrt{13}}}{2} + \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{\sqrt{1+\sqrt{13}}}, \frac{I\sqrt{3}}{6}+\frac{I\sqrt{39}}{6}\right)\sqrt{7+2\sqrt{13}}$$

Result(type 4, 199 leaves):

$$\frac{1}{\sqrt{-6+6\sqrt{13}}\,\sqrt{-x^4+x^2+3}\,\left(1+\sqrt{13}\right)}\left(36\sqrt{1-\left(-\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-6+6\sqrt{13}}}{6},\frac{I\sqrt{3}}{6}+\frac{I\sqrt{39}}{6}\right)\right.\right.$$
$$\left.\left.-\mathrm{EllipticE}\left(\frac{x\sqrt{-6+6\sqrt{13}}}{6},\frac{I\sqrt{3}}{6}+\frac{I\sqrt{39}}{6}\right)\right)\right)$$
$$+\frac{18\sqrt{1-\left(-\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+6\sqrt{13}}}{6},\frac{I\sqrt{3}}{6}+\frac{I\sqrt{39}}{6}\right)}{\sqrt{-6+6\sqrt{13}}\,\sqrt{-x^4+x^2+3}}$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int\frac{-x^2+3}{\sqrt{-x^4-2x^2+3}}\,dx$$

Optimal(type 4, 27 leaves, 4 steps):

$$-\mathrm{EllipticE}\left(x,\frac{I}{3}\sqrt{3}\right)\sqrt{3}+2\,\mathrm{EllipticF}\left(x,\frac{I}{3}\sqrt{3}\right)\sqrt{3}$$

Result(type 4, 94 leaves):

$$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(\mathrm{EllipticF}\left(x,\frac{I}{3}\sqrt{3}\right)-\mathrm{EllipticE}\left(x,\frac{I}{3}\sqrt{3}\right)\right)}{\sqrt{-x^4-2x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x,\frac{I}{3}\sqrt{3}\right)}{\sqrt{-x^4-2x^2+3}}$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int\frac{-x^2+3}{\sqrt{-x^4-3x^2+3}}\,dx$$

Optimal(type 4, 74 leaves, 4 steps):

$$-\frac{\mathrm{EllipticE}\left(\frac{x\sqrt{2}}{\sqrt{-3+\sqrt{21}}},\frac{I\sqrt{7}}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{6+2\sqrt{21}}}{2}+\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{\sqrt{-3+\sqrt{21}}},\frac{I\sqrt{7}}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{3+2\sqrt{21}}$$

Result(type 4, 203 leaves):

$$\frac{36\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{18+6\sqrt{21}}}{6},\frac{I\sqrt{7}}{2}-\frac{I\sqrt{3}}{2}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{18+6\sqrt{21}}}{6},\frac{I\sqrt{7}}{2}-\frac{I\sqrt{3}}{2}\right)\right)}{\sqrt{18+6\sqrt{21}}\,\sqrt{-x^4-3x^2+3}\,\left(-3+\sqrt{21}\right)}$$

$$+\frac{18\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{18+6\sqrt{21}}}{6},\frac{I\sqrt{7}}{2}-\frac{I\sqrt{3}}{2}\right)}{\sqrt{18+6\sqrt{21}}\,\sqrt{-x^4-3x^2+3}}$$

Problem 48: Result more than twice size of optimal antiderivative.

$$\int \frac{1+x^2\sqrt{\frac{c}{a}}}{\sqrt{c\,x^4-a}}\,dx$$

Optimal(type 4, 44 leaves, 3 steps):

$$\frac{\text{EllipticE}\left(\left(\frac{c}{a}\right)^{1/4}x, I\right)\sqrt{1-\frac{c\,x^4}{a}}}{\left(\frac{c}{a}\right)^{1/4}\sqrt{c\,x^4-a}}$$

Result(type 4, 164 leaves):

$$\frac{\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}, I\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4-a}}$$
$$+\frac{\sqrt{\frac{c}{a}}\sqrt{a}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}, I\right)-\text{EllipticE}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}, I\right)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4-a}\sqrt{c}}$$

Problem 49: Unable to integrate problem.

$$\int \frac{(b\,x^4+1)^p}{(-x^2+1)^3}\,dx$$

Optimal(type 6, 81 leaves, 6 steps):

$$x\,AppellF1\left(\frac{1}{4}, 3, -p, \frac{5}{4}, x^4, -b\,x^4\right)+x^3\,AppellF1\left(\frac{3}{4}, 3, -p, \frac{7}{4}, x^4, -b\,x^4\right)+\frac{3\,x^5\,AppellF1\left(\frac{5}{4}, 3, -p, \frac{9}{4}, x^4, -b\,x^4\right)}{5}+\frac{x^7\,AppellF1\left(\frac{7}{4}, 3, -p, \frac{11}{4}, x^4, -b\,x^4\right)}{7}$$

Result(type 8, 21 leaves):

$$\int \frac{(b x^4+1)^p}{(-x^2+1)^3}\, dx$$

Problem 51: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{e x^2+d}\,(-e^2 x^4+d^2)}\, dx$$

Optimal(type 3, 45 leaves, 4 steps):

$$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}\,\sqrt{e}}{\sqrt{e x^2+d}}\right)\sqrt{2}}{4 d^2\sqrt{e}}+\frac{x}{2 d^2\sqrt{e x^2+d}}$$

Result(type 3, 440 leaves):

$$\frac{e\sqrt{2}\,\ln\left(\frac{4 d+2\sqrt{d e}\left(x-\frac{\sqrt{d e}}{e}\right)+2\sqrt{2}\,\sqrt{d}\,\sqrt{\left(x-\frac{\sqrt{d e}}{e}\right)^2 e+2\sqrt{d e}\left(x-\frac{\sqrt{d e}}{e}\right)+2 d}}{x-\frac{\sqrt{d e}}{e}}\right)}{4\sqrt{d e}\,\left(\sqrt{-d e}+\sqrt{d e}\right)\left(-\sqrt{-d e}+\sqrt{d e}\right)\sqrt{d}}$$

$$-\frac{e\sqrt{2}\,\ln\left(\frac{4 d-2\sqrt{d e}\left(x+\frac{\sqrt{d e}}{e}\right)+2\sqrt{2}\,\sqrt{d}\,\sqrt{\left(x+\frac{\sqrt{d e}}{e}\right)^2 e-2\sqrt{d e}\left(x+\frac{\sqrt{d e}}{e}\right)+2 d}}{x+\frac{\sqrt{d e}}{e}}\right)}{4\sqrt{d e}\,\left(\sqrt{-d e}+\sqrt{d e}\right)\left(-\sqrt{-d e}+\sqrt{d e}\right)\sqrt{d}}$$

$$+\frac{\sqrt{\left(x-\frac{\sqrt{-d e}}{e}\right)^2 e+2\sqrt{-d e}\left(x-\frac{\sqrt{-d e}}{e}\right)}}{2\left(\sqrt{-d e}+\sqrt{d e}\right)\left(-\sqrt{-d e}+\sqrt{d e}\right) d\left(x-\frac{\sqrt{-d e}}{e}\right)}+\frac{\sqrt{\left(x+\frac{\sqrt{-d e}}{e}\right)^2 e-2\sqrt{-d e}\left(x+\frac{\sqrt{-d e}}{e}\right)}}{2\left(\sqrt{-d e}+\sqrt{d e}\right)\left(-\sqrt{-d e}+\sqrt{d e}\right) d\left(x+\frac{\sqrt{-d e}}{e}\right)}$$

Problem 55: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-b x^2+a}\,\sqrt{-b^2 x^4+a^2}}\, dx$$

Optimal(type 3, 62 leaves, 3 steps):

$$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}\,\sqrt{b}}{\sqrt{b x^2+a}}\right)\sqrt{-b x^2+a}\,\sqrt{b x^2+a}\,\sqrt{2}}{2 a\sqrt{b}\,\sqrt{-b^2 x^4+a^2}}$$

Result(type 3, 265 leaves):

$$\frac{1}{2\left(b x^2-a\right)\sqrt{b x^2+a}\,\left(\sqrt{a b}+\sqrt{-a b}\right)\left(-\sqrt{a b}+\sqrt{-a b}\right)\sqrt{a b}}\left(\sqrt{-b x^2+a}\,\sqrt{-b^2 x^4+a^2}\left(b\sqrt{a}\,\sqrt{2}\ln\left(\frac{2 b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{b x^2+a}+\sqrt{a b}\,x+a\right)}{b x-\sqrt{a b}}\right)\right.\right.$$
$$-b\sqrt{a}\,\sqrt{2}\ln\left(\frac{2 b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{b x^2+a}-\sqrt{a b}\,x+a\right)}{b x+\sqrt{a b}}\right)-2\sqrt{b}\ln\left(\frac{\sqrt{b x^2+a}\,\sqrt{b}+b x}{\sqrt{b}}\right)\sqrt{a b}$$
$$\left.\left.+2\sqrt{b}\ln\left(\frac{\sqrt{-\frac{\left(b x+\sqrt{-a b}\right)\left(-b x+\sqrt{-a b}\right)}{b}}\,\sqrt{b}+b x}{\sqrt{b}}\right)\sqrt{a b}\right)\right)$$

Problem 57: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(e x^2+d\right)^{5/2}}{c e^2 x^4+b e^2 x^2+b d e-c d^2}\,dx$$

Optimal(type 3, 113 leaves, 7 steps):

$$\frac{\left(-2 b e+5 c d\right)\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e x^2+d}}\right)}{2 c^2\sqrt{e}}-\frac{\left(-b e+2 c d\right)^{3/2}\operatorname{arctanh}\left(\frac{x\sqrt{e}\,\sqrt{-b e+2 c d}}{\sqrt{-b e+c d}\,\sqrt{e x^2+d}}\right)}{c^2\sqrt{e}\,\sqrt{-b e+c d}}+\frac{x\sqrt{e x^2+d}}{2 c}$$

Result(type ?, 7002 leaves): Display of huge result suppressed!

Problem 58: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(e x^2+d\right)^{3/2}}{c e^2 x^4+b e^2 x^2+b d e-c d^2}\,dx$$

Optimal(type 3, 88 leaves, 6 steps):

$$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e x^2+d}}\right)}{c\sqrt{e}}-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}\,\sqrt{-b e+2 c d}}{\sqrt{-b e+c d}\,\sqrt{e x^2+d}}\right)\sqrt{-b e+2 c d}}{c\sqrt{e}\,\sqrt{-b e+c d}}$$

Result(type ?, 4307 leaves): Display of huge result suppressed!

Problem 61: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^3}\,dx$$

Optimal(type 4, 94 leaves, 23 steps):

$$\frac{\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)}{4}+\frac{x\sqrt{x^4+x^2+1}}{4\left(x^2+1\right)^2}+\frac{\left(x^2+1\right)\sqrt{\cos\left(2\arctan(x)\right)^2}\,\mathrm{EllipticE}\left(\sin\left(2\arctan(x)\right),\frac{1}{2}\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}}{4\cos\left(2\arctan(x)\right)\sqrt{x^4+x^2+1}}$$

Result(type 4, 332 leaves):

$$\frac{x\sqrt{x^4+x^2+1}}{4\left(x^2+1\right)^2}+\frac{x\sqrt{x^4+x^2+1}}{4\left(x^2+1\right)}+\frac{\sqrt{1+\frac{x^2}{2}-\frac{\mathrm{I}x^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{\mathrm{I}x^2\sqrt{3}}{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2\mathrm{I}\sqrt{3}}}{2},\frac{\sqrt{-2+2\mathrm{I}\sqrt{3}}}{2}\right)}{\sqrt{-2+2\mathrm{I}\sqrt{3}}\sqrt{x^4+x^2+1}\left(\mathrm{I}\sqrt{3}+1\right)}$$
$$-\frac{\sqrt{1+\frac{x^2}{2}-\frac{\mathrm{I}x^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{\mathrm{I}x^2\sqrt{3}}{2}}\,\mathrm{EllipticE}\left(\frac{x\sqrt{-2+2\mathrm{I}\sqrt{3}}}{2},\frac{\sqrt{-2+2\mathrm{I}\sqrt{3}}}{2}\right)}{\sqrt{-2+2\mathrm{I}\sqrt{3}}\sqrt{x^4+x^2+1}\left(\mathrm{I}\sqrt{3}+1\right)}$$
$$+\frac{\sqrt{1+\frac{x^2}{2}-\frac{\mathrm{I}x^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{\mathrm{I}x^2\sqrt{3}}{2}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}\,x,-\frac{1}{-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}},\frac{\sqrt{-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\right)}{2\sqrt{-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}\sqrt{x^4+x^2+1}}$$

Problem 62: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^4}\,dx$$

Optimal(type 4, 172 leaves, 26 steps):

$$\frac{\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)}{4}+\frac{x\sqrt{x^4+x^2+1}}{6\left(x^2+1\right)^3}+\frac{x\sqrt{x^4+x^2+1}}{6\left(x^2+1\right)^2}+\frac{\left(x^2+1\right)\sqrt{\cos\left(2\arctan(x)\right)^2}\,\mathrm{EllipticE}\left(\sin\left(2\arctan(x)\right),\frac{1}{2}\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}}{3\cos\left(2\arctan(x)\right)\sqrt{x^4+x^2+1}}$$
$$-\frac{\left(x^2+1\right)\sqrt{\cos\left(2\arctan(x)\right)^2}\,\mathrm{EllipticF}\left(\sin\left(2\arctan(x)\right),\frac{1}{2}\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}}{8\cos\left(2\arctan(x)\right)\sqrt{x^4+x^2+1}}$$

Result(type 4, 437 leaves):

$$\frac{x\sqrt{x^4+x^2+1}}{6\left(x^2+1\right)^3}+\frac{x\sqrt{x^4+x^2+1}}{6\left(x^2+1\right)^2}+\frac{x\sqrt{x^4+x^2+1}}{3\left(x^2+1\right)}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{\mathrm{I}x^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{\mathrm{I}x^2\sqrt{3}}{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2\,\mathrm{I}\sqrt{3}}}{2},\frac{\sqrt{-2+2\,\mathrm{I}\sqrt{3}}}{2}\right)}{3\sqrt{-2+2\,\mathrm{I}\sqrt{3}}\sqrt{x^4+x^2+1}}$$

$$+\frac{4\sqrt{1+\frac{x^2}{2}-\frac{\mathrm{I}x^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{\mathrm{I}x^2\sqrt{3}}{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2\,\mathrm{I}\sqrt{3}}}{2},\frac{\sqrt{-2+2\,\mathrm{I}\sqrt{3}}}{2}\right)}{3\sqrt{-2+2\,\mathrm{I}\sqrt{3}}\sqrt{x^4+x^2+1}\left(\mathrm{I}\sqrt{3}+1\right)}$$

$$-\frac{4\sqrt{1+\frac{x^2}{2}-\frac{\mathrm{I}x^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{\mathrm{I}x^2\sqrt{3}}{2}}\,\mathrm{EllipticE}\left(\frac{x\sqrt{-2+2\,\mathrm{I}\sqrt{3}}}{2},\frac{\sqrt{-2+2\,\mathrm{I}\sqrt{3}}}{2}\right)}{3\sqrt{-2+2\,\mathrm{I}\sqrt{3}}\sqrt{x^4+x^2+1}\left(\mathrm{I}\sqrt{3}+1\right)}$$

$$+\frac{\sqrt{1+\frac{x^2}{2}-\frac{\mathrm{I}x^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{\mathrm{I}x^2\sqrt{3}}{2}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}\,x,-\frac{1}{-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}},\frac{\sqrt{-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\right)}{2\sqrt{-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}\sqrt{x^4+x^2+1}}$$

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Problem 65: Result more than twice size of optimal antiderivative.
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$$\int\frac{\left(x^2+1\right)^2}{\left(x^4+x^2+1\right)^{3/2}}\,dx$$

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Optimal(type 4, 99 leaves, 2 steps):
```

$$\frac{x\left(2x^2+1\right)}{3\sqrt{x^4+x^2+1}}-\frac{2x\sqrt{x^4+x^2+1}}{3\left(x^2+1\right)}+\frac{2\left(x^2+1\right)\sqrt{\cos(2\arctan(x))^2}\,\mathrm{EllipticE}\left(\sin(2\arctan(x)),\frac{1}{2}\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}}{3\cos(2\arctan(x))\sqrt{x^4+x^2+1}}$$

```
Result(type 4, 267 leaves):
```

$$-\frac{2\left(-\frac{1}{6}x+\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2\,\mathrm{I}\sqrt{3}}}{2},\frac{\sqrt{-2+2\,\mathrm{I}\sqrt{3}}}{2}\right)}{3\sqrt{-2+2\,\mathrm{I}\sqrt{3}}\sqrt{x^4+x^2+1}}$$

$$+\frac{1}{3\sqrt{-2+2\,\mathrm{I}\sqrt{3}}\sqrt{x^4+x^2+1}\left(\mathrm{I}\sqrt{3}+1\right)}\left(8\sqrt{1-\left(-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2\,\mathrm{I}\sqrt{3}}}{2},\right.\right.\right.$$

$$\left.\left.\left.\frac{\sqrt{-2+2 I\sqrt{3}}}{2}\right) - \text{EllipticE}\left(\frac{x\sqrt{-2+2 I\sqrt{3}}}{2}, \frac{\sqrt{-2+2 I\sqrt{3}}}{2}\right)\right)\right) - \frac{2\left(\frac{1}{6}x^3+\frac{1}{3}x\right)}{\sqrt{x^4+x^2+1}} - \frac{4\left(-\frac{1}{3}x^3-\frac{1}{6}x\right)}{\sqrt{x^4+x^2+1}}$$

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Problem 66: Result more than twice size of optimal antiderivative.
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$$\int \frac{1}{(x^2+1)^2 (x^4+x^2+1)^{3/2}}\, dx$$

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Optimal(type 4, 110 leaves, 16 steps):
```

$$\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) - \frac{x(x^2+2)}{3\sqrt{x^4+x^2+1}} + \frac{x\sqrt{x^4+x^2+1}}{3(x^2+1)} + \frac{(x^2+1)\sqrt{\cos(2\arctan(x))^2}\,\text{EllipticE}\left(\sin(2\arctan(x)), \frac{1}{2}\right)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}}{6\cos(2\arctan(x))\sqrt{x^4+x^2+1}}$$

```
Result(type 4, 418 leaves):
```

$$-\frac{2\left(\frac{1}{6}x^3+\frac{1}{3}x\right)}{\sqrt{x^4+x^2+1}} + \frac{x\sqrt{x^4+x^2+1}}{2(x^2+1)} - \frac{5\sqrt{1+\frac{x^2}{2}-\frac{I x^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{I x^2\sqrt{3}}{2}}\,\text{EllipticF}\left(\frac{x\sqrt{-2+2 I\sqrt{3}}}{2}, \frac{\sqrt{-2+2 I\sqrt{3}}}{2}\right)}{3\sqrt{-2+2 I\sqrt{3}}\sqrt{x^4+x^2+1}}$$

$$+\frac{2\sqrt{1+\frac{x^2}{2}-\frac{I x^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{I x^2\sqrt{3}}{2}}\,\text{EllipticF}\left(\frac{x\sqrt{-2+2 I\sqrt{3}}}{2}, \frac{\sqrt{-2+2 I\sqrt{3}}}{2}\right)}{3\sqrt{-2+2 I\sqrt{3}}\sqrt{x^4+x^2+1}\,(I\sqrt{3}+1)}$$

$$-\frac{2\sqrt{1+\frac{x^2}{2}-\frac{I x^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{I x^2\sqrt{3}}{2}}\,\text{EllipticE}\left(\frac{x\sqrt{-2+2 I\sqrt{3}}}{2}, \frac{\sqrt{-2+2 I\sqrt{3}}}{2}\right)}{3\sqrt{-2+2 I\sqrt{3}}\sqrt{x^4+x^2+1}\,(I\sqrt{3}+1)}$$

$$+\frac{2\sqrt{1+\frac{x^2}{2}-\frac{I x^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{I x^2\sqrt{3}}{2}}\,\text{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{I\sqrt{3}}{2}}\,x, -\frac{1}{-\frac{1}{2}+\frac{I\sqrt{3}}{2}}, \frac{\sqrt{-\frac{1}{2}-\frac{I\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{I\sqrt{3}}{2}}}\right)}{\sqrt{-\frac{1}{2}+\frac{I\sqrt{3}}{2}}\sqrt{x^4+x^2+1}}$$

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Problem 67: Result more than twice size of optimal antiderivative.
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$$\int \frac{1}{(x^2+1)^3 (x^4+x^2+1)^{3/2}}\, dx$$

Optimal(type 4, 192 leaves, 23 steps):

$$\frac{3\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)}{4}-\frac{x\left(-x^2+1\right)}{3\sqrt{x^4+x^2+1}}+\frac{x\sqrt{x^4+x^2+1}}{4\left(x^2+1\right)^2}-\frac{x\sqrt{x^4+x^2+1}}{3\left(x^2+1\right)}$$
$$+\frac{19\left(x^2+1\right)\sqrt{\cos(2\arctan(x))^2}\,\mathrm{EllipticE}\left(\sin(2\arctan(x)),\frac{1}{2}\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}}{12\cos(2\arctan(x))\sqrt{x^4+x^2+1}}$$
$$-\frac{5\left(x^2+1\right)\sqrt{\cos(2\arctan(x))^2}\,\mathrm{EllipticF}\left(\sin(2\arctan(x)),\frac{1}{2}\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}}{4\cos(2\arctan(x))\sqrt{x^4+x^2+1}}$$

Result(type 4, 438 leaves):

$$-\frac{2\left(\frac{1}{6}x-\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}}+\frac{x\sqrt{x^4+x^2+1}}{4\left(x^2+1\right)^2}+\frac{5x\sqrt{x^4+x^2+1}}{4\left(x^2+1\right)}$$
$$-\frac{10\sqrt{1+\frac{x^2}{2}-\frac{\mathrm{I}x^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{\mathrm{I}x^2\sqrt{3}}{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2\mathrm{I}\sqrt{3}}}{2},\frac{\sqrt{-2+2\mathrm{I}\sqrt{3}}}{2}\right)}{3\sqrt{-2+2\mathrm{I}\sqrt{3}}\sqrt{x^4+x^2+1}}$$
$$+\frac{19\sqrt{1+\frac{x^2}{2}-\frac{\mathrm{I}x^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{\mathrm{I}x^2\sqrt{3}}{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2\mathrm{I}\sqrt{3}}}{2},\frac{\sqrt{-2+2\mathrm{I}\sqrt{3}}}{2}\right)}{3\sqrt{-2+2\mathrm{I}\sqrt{3}}\sqrt{x^4+x^2+1}\left(\mathrm{I}\sqrt{3}+1\right)}$$
$$-\frac{19\sqrt{1+\frac{x^2}{2}-\frac{\mathrm{I}x^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{\mathrm{I}x^2\sqrt{3}}{2}}\,\mathrm{EllipticE}\left(\frac{x\sqrt{-2+2\mathrm{I}\sqrt{3}}}{2},\frac{\sqrt{-2+2\mathrm{I}\sqrt{3}}}{2}\right)}{3\sqrt{-2+2\mathrm{I}\sqrt{3}}\sqrt{x^4+x^2+1}\left(\mathrm{I}\sqrt{3}+1\right)}$$
$$+\frac{3\sqrt{1+\frac{x^2}{2}-\frac{\mathrm{I}x^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{\mathrm{I}x^2\sqrt{3}}{2}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}\,x,-\frac{1}{-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}},\frac{\sqrt{-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\right)}{2\sqrt{-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}\sqrt{x^4+x^2+1}}$$

Problem 70: Result more than twice size of optimal antiderivative.

$$\int \frac{(c x^4 + b x^2 + a)^2}{(e x^2 + d)^3}\, dx$$

Optimal(type 3, 185 leaves, 5 steps):

$$-\frac{c(-2 b e + 3 c d) x}{e^4} + \frac{c^2 x^3}{3 e^3} + \frac{(a e^2 - b d e + c d^2)^2 x}{4 d e^4 (e x^2 + d)^2} - \frac{(-3 a e^2 - 5 b d e + 13 c d^2)(a e^2 - b d e + c d^2) x}{8 d^2 e^4 (e x^2 + d)}$$
$$+ \frac{(35 c^2 d^4 - 6 c d^2 e(-a e + 5 b d) + e^2(3 e^2 a^2 + 2 a b d e + 3 b^2 d^2)) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{8 d^{5/2} e^{9/2}}$$

Result(type 3, 401 leaves):

$$\frac{c^2 x^3}{3 e^3} + \frac{2 c b x}{e^3} - \frac{3 c^2 x d}{e^4} + \frac{3 e x^3 a^2}{8 (e x^2 + d)^2 d^2} + \frac{x^3 a b}{4 (e x^2 + d)^2 d} - \frac{5 x^3 a c}{4 e (e x^2 + d)^2} - \frac{5 x^3 b^2}{8 e (e x^2 + d)^2} + \frac{9 d x^3 b c}{4 e^2 (e x^2 + d)^2} - \frac{13 d^2 x^3 c^2}{8 e^3 (e x^2 + d)^2}$$
$$+ \frac{5 x a^2}{8 (e x^2 + d)^2 d} - \frac{x a b}{4 e (e x^2 + d)^2} - \frac{3 d x a c}{4 e^2 (e x^2 + d)^2} - \frac{3 d x b^2}{8 e^2 (e x^2 + d)^2} + \frac{7 d^2 x b c}{4 e^3 (e x^2 + d)^2} - \frac{11 d^3 x c^2}{8 e^4 (e x^2 + d)^2} + \frac{3 \arctan\left(\frac{e x}{\sqrt{d e}}\right) a^2}{8 d^2 \sqrt{d e}}$$
$$+ \frac{\arctan\left(\frac{e x}{\sqrt{d e}}\right) a b}{4 e d \sqrt{d e}} + \frac{3 \arctan\left(\frac{e x}{\sqrt{d e}}\right) a c}{4 e^2 \sqrt{d e}} + \frac{3 \arctan\left(\frac{e x}{\sqrt{d e}}\right) b^2}{8 e^2 \sqrt{d e}} - \frac{15 d \arctan\left(\frac{e x}{\sqrt{d e}}\right) b c}{4 e^3 \sqrt{d e}} + \frac{35 d^2 \arctan\left(\frac{e x}{\sqrt{d e}}\right) c^2}{8 e^4 \sqrt{d e}}$$

Problem 71: Result more than twice size of optimal antiderivative.

$$\int \frac{(c x^4 + b x^2 + a)^2}{(e x^2 + d)^4}\, dx$$

Optimal(type 3, 234 leaves, 5 steps):

$$\frac{c^2 x}{e^4} + \frac{(a e^2 - b d e + c d^2)^2 x}{6 d e^4 (e x^2 + d)^3} - \frac{(-5 a e^2 - 7 b d e + 19 c d^2)(a e^2 - b d e + c d^2) x}{24 d^2 e^4 (e x^2 + d)^2}$$
$$+ \frac{(29 c^2 d^4 - 2 c d^2 e(-a e + 11 b d) + e^2(5 e^2 a^2 + 2 a b d e + b^2 d^2)) x}{16 d^3 e^4 (e x^2 + d)}$$
$$- \frac{(35 c^2 d^4 - 2 c d^2 e(a e + 5 b d) - e^2(5 e^2 a^2 + 2 a b d e + b^2 d^2)) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{16 d^{7/2} e^{9/2}}$$

Result(type 3, 505 leaves):

$$\frac{5 e^2 x^5 a^2}{16 (e x^2 + d)^3 d^3} - \frac{11 x^5 b c}{8 e (e x^2 + d)^3} + \frac{29 d x^5 c^2}{16 e^2 (e x^2 + d)^3} + \frac{5 e x^3 a^2}{6 (e x^2 + d)^3 d^2} - \frac{x^3 a c}{3 e (e x^2 + d)^3} + \frac{17 d^2 x^3 c^2}{6 e^3 (e x^2 + d)^3} - \frac{x a b}{8 e (e x^2 + d)^3} - \frac{d x b^2}{16 e^2 (e x^2 + d)^3}$$

$$+\frac{19 d^3 x c^2}{16 e^4 (e x^2+d)^3}+\frac{\arctan\left(\frac{e x}{\sqrt{d e}}\right) b^2}{16 e^2 d \sqrt{d e}}+\frac{5 \arctan\left(\frac{e x}{\sqrt{d e}}\right) b c}{8 e^3 \sqrt{d e}}-\frac{35 d \arctan\left(\frac{e x}{\sqrt{d e}}\right) c^2}{16 e^4 \sqrt{d e}}+\frac{x^5 a c}{8 (e x^2+d)^3 d}+\frac{x^3 a b}{3 (e x^2+d)^3 d}+\frac{c^2 x}{e^4}$$

$$-\frac{5 d^2 x b c}{8 e^3 (e x^2+d)^3}+\frac{\arctan\left(\frac{e x}{\sqrt{d e}}\right) a b}{8 e d^2 \sqrt{d e}}+\frac{\arctan\left(\frac{e x}{\sqrt{d e}}\right) a c}{8 e^2 d \sqrt{d e}}+\frac{e x^5 a b}{8 (e x^2+d)^3 d^2}-\frac{5 d x^3 b c}{3 e^2 (e x^2+d)^3}-\frac{d x a c}{8 e^2 (e x^2+d)^3}+\frac{x^5 b^2}{16 (e x^2+d)^3 d}$$

$$+\frac{11 x a^2}{16 (e x^2+d)^3 d}-\frac{x^3 b^2}{6 e (e x^2+d)^3}+\frac{5 \arctan\left(\frac{e x}{\sqrt{d e}}\right) a^2}{16 d^3 \sqrt{d e}}$$

Problem 72: Result more than twice size of optimal antiderivative.

$$\int \frac{(e x^2+d)^3}{c x^4+b x^2+a}\, dx$$

Optimal(type 3, 280 leaves, 5 steps):

$$\frac{e^2 (-b e+3 c d) x}{c^2}+\frac{e^3 x^3}{3 c}$$

$$+\frac{\arctan\left(\frac{x \sqrt{2} \sqrt{c}}{\sqrt{b-\sqrt{-4 a c+b^2}}}\right)\left(e\left(3 c^2 d^2+b^2 e^2-c e (a e+3 b d)\right)+\frac{(-b e+2 c d)\left(c^2 d^2+b^2 e^2-c e (3 a e+b d)\right)}{\sqrt{-4 a c+b^2}}\right)\sqrt{2}}{2 c^{5/2} \sqrt{b-\sqrt{-4 a c+b^2}}}$$

$$+\frac{\arctan\left(\frac{x \sqrt{2} \sqrt{c}}{\sqrt{b+\sqrt{-4 a c+b^2}}}\right)\left(e\left(3 c^2 d^2+b^2 e^2-c e (a e+3 b d)\right)-\frac{(-b e+2 c d)\left(c^2 d^2+b^2 e^2-c e (3 a e+b d)\right)}{\sqrt{-4 a c+b^2}}\right)\sqrt{2}}{2 c^{5/2} \sqrt{b+\sqrt{-4 a c+b^2}}}$$

Result(type 3, 1210 leaves):

$$\frac{e^3 x^3}{3 c}-\frac{e^3 b x}{c^2}+\frac{3 e^2 x d}{c}+\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) a e^3}{2 c \sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) b^2 e^3}{2 c^2 \sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}$$

$$+\frac{3 \sqrt{2}\, \operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) b d e^2}{2 c \sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}-\frac{3 \sqrt{2}\, \operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) d^2 e}{2 \sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}$$

$$- \frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) a b e^3}{2 c \sqrt{-4 a c+b^2}\,\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}} + \frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) a d e^2}{\sqrt{-4 a c+b^2}\,\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}$$

$$+ \frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) b^3 e^3}{2 c^2 \sqrt{-4 a c+b^2}\,\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}} - \frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) b^2 d e^2}{2 c \sqrt{-4 a c+b^2}\,\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}$$

$$+ \frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) d^2 e b}{2 \sqrt{-4 a c+b^2}\,\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}} - \frac{c\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) d^3}{\sqrt{-4 a c+b^2}\,\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}} - \frac{\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) a e^3}{2 c \sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}$$

$$+ \frac{\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) b^2 e^3}{2 c^2 \sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}} - \frac{3\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) b d e^2}{2 c \sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}} + \frac{3\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) d^2 e}{2 \sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}$$

$$- \frac{3\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) a b e^3}{2 c \sqrt{-4 a c+b^2}\,\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}} + \frac{3\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) a d e^2}{\sqrt{-4 a c+b^2}\,\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}} + \frac{\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) b^3 e^3}{2 c^2 \sqrt{-4 a c+b^2}\,\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}$$

$$- \frac{3\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) b^2 d e^2}{2 c \sqrt{-4 a c+b^2}\,\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}} + \frac{3\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) d^2 e b}{2 \sqrt{-4 a c+b^2}\,\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}} - \frac{c\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) d^3}{\sqrt{-4 a c+b^2}\,\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}$$

Problem 85: Result more than twice size of optimal antiderivative.

$$\int (5 x^2+7) (-x^4+x^2+2)^{3/2}\, dx$$

Optimal(type 4, 75 leaves, 6 steps):

$$\frac{x (35 x^2+48) (-x^4+x^2+2)^{3/2}}{63} + \frac{4432\, \mathrm{EllipticE}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)}{315} + \frac{418\, \mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)}{105} + \frac{x (669 x^2+1087) \sqrt{-x^4+x^2+2}}{315}$$

Result(type 4, 175 leaves):

$$-\frac{13 x^5 \sqrt{-x^4+x^2+2}}{63}+\frac{1259 x^3 \sqrt{-x^4+x^2+2}}{315}+\frac{1567 x \sqrt{-x^4+x^2+2}}{315}+\frac{2843 \sqrt{2}\, \sqrt{-2 x^2+4}\, \sqrt{x^2+1}\ \mathrm{EllipticF}\left(\frac{x \sqrt{2}}{2}, \mathrm{I} \sqrt{2}\right)}{315 \sqrt{-x^4+x^2+2}}$$
$$-\frac{2216 \sqrt{2}\, \sqrt{-2 x^2+4}\, \sqrt{x^2+1}\left(\mathrm{EllipticF}\left(\frac{x \sqrt{2}}{2}, \mathrm{I} \sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{x \sqrt{2}}{2}, \mathrm{I} \sqrt{2}\right)\right)}{315 \sqrt{-x^4+x^2+2}}-\frac{5 x^7 \sqrt{-x^4+x^2+2}}{9}$$

```
Problem 86: Result more than twice size of optimal antiderivative.
```

$$\int \frac{\left(-x^4+x^2+2\right)^{3/2}}{\left(5 x^2+7\right)^2}\, dx$$

```
Optimal(type 4, 86 leaves, 21 steps):
```

$$-\frac{97\, \mathrm{EllipticE}\left(\frac{x \sqrt{2}}{2}, \mathrm{I} \sqrt{2}\right)}{525}+\frac{458\, \mathrm{EllipticF}\left(\frac{x \sqrt{2}}{2}, \mathrm{I} \sqrt{2}\right)}{875}-\frac{1241\, \mathrm{EllipticPi}\left(\frac{x \sqrt{2}}{2},-\frac{10}{7}, \mathrm{I} \sqrt{2}\right)}{6125}-\frac{x \sqrt{-x^4+x^2+2}}{75}-\frac{17 x \sqrt{-x^4+x^2+2}}{175\left(5 x^2+7\right)}$$

```
Result(type 4, 179 leaves):
```

$$-\frac{17 x \sqrt{-x^4+x^2+2}}{175\left(5 x^2+7\right)}-\frac{x \sqrt{-x^4+x^2+2}}{75}+\frac{229 \sqrt{2}\, \sqrt{-2 x^2+4}\, \sqrt{x^2+1}\ \mathrm{EllipticF}\left(\frac{x \sqrt{2}}{2}, \mathrm{I} \sqrt{2}\right)}{875 \sqrt{-x^4+x^2+2}}-\frac{97 \sqrt{2}\, \sqrt{-2 x^2+4}\, \sqrt{x^2+1}\ \mathrm{EllipticE}\left(\frac{x \sqrt{2}}{2}, \mathrm{I} \sqrt{2}\right)}{1050 \sqrt{-x^4+x^2+2}}$$
$$-\frac{1241 \sqrt{2}\, \sqrt{1-\frac{x^2}{2}}\, \sqrt{x^2+1}\ \mathrm{EllipticPi}\left(\frac{x \sqrt{2}}{2},-\frac{10}{7}, \mathrm{I} \sqrt{2}\right)}{6125 \sqrt{-x^4+x^2+2}}$$

```
Problem 87: Result more than twice size of optimal antiderivative.
```

$$\int \frac{\left(5 x^2+7\right)^3}{\sqrt{-x^4+x^2+2}}\, dx$$

```
Optimal(type 4, 63 leaves, 6 steps):
```

$$\frac{3905\, \mathrm{EllipticE}\left(\frac{x \sqrt{2}}{2}, \mathrm{I} \sqrt{2}\right)}{3}-542\, \mathrm{EllipticF}\left(\frac{x \sqrt{2}}{2}, \mathrm{I} \sqrt{2}\right)-\frac{625 x \sqrt{-x^4+x^2+2}}{3}-25 x^3 \sqrt{-x^4+x^2+2}$$

```
Result(type 4, 141 leaves):
```

$$\frac{2279 \sqrt{2}\, \sqrt{-2 x^2+4}\, \sqrt{x^2+1}\ \mathrm{EllipticF}\left(\frac{x \sqrt{2}}{2}, \mathrm{I} \sqrt{2}\right)}{6 \sqrt{-x^4+x^2+2}}-\frac{3905 \sqrt{2}\, \sqrt{-2 x^2+4}\, \sqrt{x^2+1}\left(\mathrm{EllipticF}\left(\frac{x \sqrt{2}}{2}, \mathrm{I} \sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{x \sqrt{2}}{2}, \mathrm{I} \sqrt{2}\right)\right)}{6 \sqrt{-x^4+x^2+2}}$$

$$-\frac{625\,x\sqrt{-x^4+x^2+2}}{3}-25\,x^3\sqrt{-x^4+x^2+2}$$

Problem 88: Result more than twice size of optimal antiderivative.

$$\int \frac{(5\,x^2+7)^2}{\sqrt{-x^4+x^2+2}}\,dx$$

Optimal(type 4, 46 leaves, 5 steps):

$$\frac{260\,\text{EllipticE}\left(\frac{x\sqrt{2}}{2},\,I\sqrt{2}\right)}{3}-21\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2},\,I\sqrt{2}\right)-\frac{25\,x\sqrt{-x^4+x^2+2}}{3}$$

Result(type 4, 124 leaves):

$$\frac{197\sqrt{2}\,\sqrt{-2\,x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2},\,I\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}}-\frac{130\sqrt{2}\,\sqrt{-2\,x^2+4}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{x\sqrt{2}}{2},\,I\sqrt{2}\right)-\text{EllipticE}\left(\frac{x\sqrt{2}}{2},\,I\sqrt{2}\right)\right)}{3\sqrt{-x^4+x^2+2}}$$
$$-\frac{25\,x\sqrt{-x^4+x^2+2}}{3}$$

Problem 89: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}}\,dx$$

Optimal(type 4, 13 leaves, 2 steps):

$$\text{EllipticF}\left(\frac{x\sqrt{2}}{2},\,I\sqrt{2}\right)$$

Result(type 4, 46 leaves):

$$\frac{\sqrt{2}\,\sqrt{-2\,x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2},\,I\sqrt{2}\right)}{2\sqrt{-x^4+x^2+2}}$$

Problem 90: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(5\,x^2+7)^2\sqrt{-x^4+x^2+2}}\,dx$$

Optimal(type 4, 71 leaves, 8 steps):

$$-\frac{5\,\mathrm{EllipticE}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)}{476} - \frac{\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)}{238} + \frac{167\,\mathrm{EllipticPi}\left(\frac{x\sqrt{2}}{2}, -\frac{10}{7}, I\sqrt{2}\right)}{3332} - \frac{25\,x\sqrt{-x^4+x^2+2}}{476\,(5\,x^2+7)}$$

Result(type 4, 164 leaves):

$$-\frac{25\,x\sqrt{-x^4+x^2+2}}{476\,(5\,x^2+7)} - \frac{\sqrt{2}\,\sqrt{-2\,x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)}{476\sqrt{-x^4+x^2+2}} - \frac{5\sqrt{2}\,\sqrt{-2\,x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticE}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)}{952\sqrt{-x^4+x^2+2}}$$
$$+ \frac{167\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\mathrm{EllipticPi}\left(\frac{x\sqrt{2}}{2}, -\frac{10}{7}, I\sqrt{2}\right)}{3332\sqrt{-x^4+x^2+2}}$$

Problem 91: Result more than twice size of optimal antiderivative.

$$\int \frac{(5\,x^2+7)^5}{(-x^4+x^2+2)^{3/2}}\,dx$$

Optimal(type 4, 85 leaves, 7 steps):

$$-\frac{3482293\,\mathrm{EllipticE}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)}{18} + \frac{627857\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)}{6} + \frac{x\,(1419793\,x^2+1419985)}{18\sqrt{-x^4+x^2+2}} + \frac{27500\,x\sqrt{-x^4+x^2+2}}{3} + 625\,x^3\sqrt{-x^4+x^2+2}$$

Result(type 4, 279 leaves):

$$\frac{33614\left(\frac{5}{36}\,x - \frac{1}{36}\,x^3\right)}{\sqrt{-x^4+x^2+2}} - \frac{799361\sqrt{2}\,\sqrt{-2\,x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}$$
$$+ \frac{3482293\sqrt{2}\,\sqrt{-2\,x^2+4}\,\sqrt{x^2+1}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right) - \mathrm{EllipticE}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}} + \frac{120050\left(\frac{1}{9}\,x^3 - \frac{1}{18}\,x\right)}{\sqrt{-x^4+x^2+2}} + \frac{171500\left(\frac{1}{18}\,x^3 + \frac{2}{9}\,x\right)}{\sqrt{-x^4+x^2+2}}$$
$$+ \frac{122500\left(\frac{5}{18}\,x^3 + \frac{1}{9}\,x\right)}{\sqrt{-x^4+x^2+2}} + \frac{43750\left(\frac{7}{18}\,x^3 + \frac{5}{9}\,x\right)}{\sqrt{-x^4+x^2+2}} + \frac{27500\,x\sqrt{-x^4+x^2+2}}{3} + \frac{6250\left(\frac{17}{18}\,x^3 + \frac{7}{9}\,x\right)}{\sqrt{-x^4+x^2+2}} + 625\,x^3\sqrt{-x^4+x^2+2}$$

Problem 92: Result more than twice size of optimal antiderivative.

$$\int \frac{5\,x^2+7}{(-x^4+x^2+2)^{3/2}}\,dx$$

Optimal(type 4, 53 leaves, 5 steps):

$$-\frac{13\,\text{EllipticE}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)}{18}+\frac{17\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)}{6}+\frac{x\,(13\,x^2+25)}{18\sqrt{-x^4+x^2+2}}$$

Result(type 4, 155 leaves):

$$\frac{14\left(\frac{5}{36}\,x-\frac{1}{36}\,x^3\right)}{\sqrt{-x^4+x^2+2}}+\frac{19\sqrt{2}\,\sqrt{-2\,x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}$$
$$+\frac{13\sqrt{2}\,\sqrt{-2\,x^2+4}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)-\text{EllipticE}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}+\frac{10\left(\frac{1}{9}\,x^3-\frac{1}{18}\,x\right)}{\sqrt{-x^4+x^2+2}}$$

Problem 93: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{(5\,x^2+7)\,(-x^4+x^2+2)^{3/2}}\,dx$$

Optimal(type 4, 69 leaves, 8 steps):

$$\frac{8\,\text{EllipticE}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)}{153}+\frac{\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)}{102}-\frac{25\,\text{EllipticPi}\left(\frac{x\sqrt{2}}{2}, -\frac{10}{7}, I\sqrt{2}\right)}{238}+\frac{x\,(-16\,x^2+35)}{306\sqrt{-x^4+x^2+2}}$$

Result(type 4, 163 leaves):

$$\frac{2\left(-\frac{4}{153}\,x^3+\frac{35}{612}\,x\right)}{\sqrt{-x^4+x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2\,x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)}{204\sqrt{-x^4+x^2+2}}+\frac{4\sqrt{2}\,\sqrt{-2\,x^2+4}\,\sqrt{x^2+1}\,\text{EllipticE}\left(\frac{x\sqrt{2}}{2}, I\sqrt{2}\right)}{153\sqrt{-x^4+x^2+2}}$$
$$-\frac{25\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\text{EllipticPi}\left(\frac{x\sqrt{2}}{2}, -\frac{10}{7}, I\sqrt{2}\right)}{238\sqrt{-x^4+x^2+2}}$$

Problem 102: Result more than twice size of optimal antiderivative.

$$\int\frac{(e\,x^2+d)^3}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Optimal(type 4, 455 leaves, 5 steps):

$$\frac{e^2\,(-4\,b\,e+15\,c\,d)\,x\sqrt{c\,x^4+b\,x^2+a}}{15\,c^2}+\frac{e^3\,x^3\sqrt{c\,x^4+b\,x^2+a}}{5\,c}+\frac{e\,(45\,c^2\,d^2+8\,b^2\,e^2-3\,c\,e\,(3\,a\,e+10\,b\,d))\,x\sqrt{c\,x^4+b\,x^2+a}}{15\,c^{5/2}\,(\sqrt{a}+x^2\sqrt{c})}$$

$$-\frac{1}{15\cos\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)c^{11/4}\sqrt{cx^4+bx^2+a}}\left(a^{1/4}e\left(45c^2d^2+8b^2e^2-3ce(3ae\right.\right.$$

$$\left.\left.+10bd\right)\right)\sqrt{\cos\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)^2}\,\mathrm{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\sqrt{c}}}}{2}\right)\left(\sqrt{a}+x^2\sqrt{c}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{a}+x^2\sqrt{c}\right)^2}}\Bigg)$$

$$+\frac{1}{30\cos\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)c^{11/4}\sqrt{cx^4+bx^2+a}}\left(a^{1/4}\sqrt{\cos\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)^2}\,\mathrm{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\sqrt{c}}}}{2}\right)\left(\sqrt{a}\right.\right.$$

$$\left.\left.+x^2\sqrt{c}\right)\left(e\left(45c^2d^2+8b^2e^2-3ce(3ae+10bd)\right)+\frac{\left(4abe^3-15acde^2+15c^2d^3\right)\sqrt{c}}{\sqrt{a}}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{a}+x^2\sqrt{c}\right)^2}}\right)$$

Result(type 4, 1185 leaves):

$$\frac{d^3\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}$$

$$+e^3\left(\frac{x^3\sqrt{cx^4+bx^2+a}}{5c}-\frac{4bx\sqrt{cx^4+bx^2+a}}{15c^2}\right.$$

$$+\frac{1}{15c^2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}\left(ba\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(x\right.\right.\right.$$

$$\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4\,a\,c+b^2}}{a}}\Bigg), \frac{\sqrt{-4+\frac{2\,b\,\left(b+\sqrt{-4\,a\,c+b^2}\right)}{a\,c}}}{2}\Bigg)\Bigg) - \frac{1}{2\,\sqrt{\frac{-b+\sqrt{-4\,a\,c+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4\,a\,c+b^2}\right)}\Bigg(\Bigg(-\frac{3\,a}{5\,c}$$

$$+\frac{8\,b^2}{15\,c^2}\Bigg)\,a\,\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4\,a\,c+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4\,a\,c+b^2}\right)x^2}{a}}\,\Bigg(\mathrm{EllipticF}\Bigg(\frac{x\,\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4\,a\,c+b^2}}{a}}}{2},$$

$$\frac{\sqrt{-4+\frac{2\,b\,\left(b+\sqrt{-4\,a\,c+b^2}\right)}{a\,c}}}{2}\Bigg) - \mathrm{EllipticE}\Bigg(\frac{x\,\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4\,a\,c+b^2}}{a}}}{2}, \frac{\sqrt{-4+\frac{2\,b\,\left(b+\sqrt{-4\,a\,c+b^2}\right)}{a\,c}}}{2}\Bigg)\Bigg)\Bigg)\Bigg)$$

$$+3\,d\,e^2\Bigg(\frac{x\,\sqrt{c\,x^4+b\,x^2+a}}{3\,c}$$

$$-\frac{1}{12\,c\,\sqrt{\frac{-b+\sqrt{-4\,a\,c+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}\Bigg(a\,\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4\,a\,c+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4\,a\,c+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\Bigg(\frac{1}{2}\Bigg(x$$

$$\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4\,a\,c+b^2}}{a}}\Bigg), \frac{\sqrt{-4+\frac{2\,b\,\left(b+\sqrt{-4\,a\,c+b^2}\right)}{a\,c}}}{2}\Bigg)\Bigg)$$

$$+\frac{1}{3\,c\,\sqrt{\frac{-b+\sqrt{-4\,a\,c+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4\,a\,c+b^2}\right)}\Bigg(b\,a\,\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4\,a\,c+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4\,a\,c+b^2}\right)x^2}{a}}$$

$$\Bigg(\mathrm{EllipticF}\Bigg(\frac{x\,\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4\,a\,c+b^2}}{a}}}{2}, \frac{\sqrt{-4+\frac{2\,b\,\left(b+\sqrt{-4\,a\,c+b^2}\right)}{a\,c}}}{2}\Bigg) - \mathrm{EllipticE}\Bigg(\frac{x\,\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4\,a\,c+b^2}}{a}}}{2},$$

$$\left.\left.\left.\left.\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)\right)\right)\right)$$

$$-\frac{1}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}\left(b+\sqrt{-4ac+b^2}\right)}\left(3d^2ea\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\right.$$

$$\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\right.\right.$$

$$\left.\left.\left.\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)\right)\right)$$

Problem 108: Unable to integrate problem.

$$\int\left(bx^4+cx^2+a\right)^p\,dx$$

Optimal(type 6, 121 leaves, 2 steps):

$$\frac{x\left(bx^4+cx^2+a\right)^p \mathit{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)}{\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^p\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^p}$$

Result(type 8, 16 leaves):

$$\int\left(bx^4+cx^2+a\right)^p\,dx$$

Test results for the 109 problems in "1.2.2.4 (f x)^m (d+e x^2)^q (a+b x^2+c x^4)^p.txt"

Problem 18: Result more than twice size of optimal antiderivative.

$$\int x^5\left(ex^2+d\right)\left(x^4+2x^2+1\right)^5\,dx$$

Optimal(type 1, 55 leaves, 4 steps):

$$\frac{(d-e)\left(x^2+1\right)^{11}}{22}-\frac{(2d-3e)\left(x^2+1\right)^{12}}{24}+\frac{(d-3e)\left(x^2+1\right)^{13}}{26}+\frac{e\left(x^2+1\right)^{14}}{28}$$

Result(type 1, 129 leaves):

$$\frac{e x^{28}}{28}+\frac{(d+10 e) x^{26}}{26}+\frac{(10 d+45 e) x^{24}}{24}+\frac{(45 d+120 e) x^{22}}{22}+\frac{(120 d+210 e) x^{20}}{20}+\frac{(210 d+252 e) x^{18}}{18}+\frac{(252 d+210 e) x^{16}}{16}+\frac{(210 d+120 e) x^{14}}{14}+\frac{(120 d+45 e) x^{12}}{12}+\frac{(45 d+10 e) x^{10}}{10}+\frac{(10 d+e) x^{8}}{8}+\frac{d x^{6}}{6}$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int (f x)^m \left(x^2+1\right)\left(x^4+2x^2+1\right)^5 \,\mathrm{d}x$$

Optimal(type 3, 203 leaves, 3 steps):

$$\frac{(f x)^{1+m}}{f(1+m)}+\frac{11 (f x)^{3+m}}{f^3 (3+m)}+\frac{55 (f x)^{5+m}}{f^5 (5+m)}+\frac{165 (f x)^{7+m}}{f^7 (7+m)}+\frac{330 (f x)^{9+m}}{f^9 (9+m)}+\frac{462 (f x)^{11+m}}{f^{11} (11+m)}+\frac{462 (f x)^{13+m}}{f^{13} (13+m)}+\frac{330 (f x)^{15+m}}{f^{15} (15+m)}+\frac{165 (f x)^{17+m}}{f^{17} (17+m)}+\frac{55 (f x)^{19+m}}{f^{19} (19+m)}+\frac{11 (f x)^{21+m}}{f^{21} (21+m)}+\frac{(f x)^{23+m}}{f^{23} (23+m)}$$

Result(type 3, 1120 leaves):

$$\Big((f x)^m \big(m^{11} x^{22}+121 m^{10} x^{22}+11 m^{11} x^{20}+6435 m^9 x^{22}+1353 m^{10} x^{20}+197835 m^8 x^{22}+55 m^{11} x^{18}+72985 m^9 x^{20}+3889578 m^7 x^{22}+6875 m^{10} x^{18}+2271555 m^8 x^{20}+51069018 m^6 x^{22}+165 m^{11} x^{16}+376365 m^9 x^{18}+45134958 m^7 x^{20}+453714470 m^5 x^{22}+20955 m^{10} x^{16}+11870265 m^8 x^{18}+597988314 m^6 x^{20}+2702025590 m^4 x^{22}+330 m^{11} x^{14}+1164735 m^9 x^{16}+238653030 m^7 x^{18}+5353566130 m^5 x^{20}+10431670821 m^3 x^{22}+42570 m^{10} x^{14}+37263105 m^8 x^{16}+3194704590 m^6 x^{18}+32087153670 m^4 x^{20}+24372200061 m^2 x^{22}+462 m^{11} x^{12}+2403390 m^9 x^{14}+759091410 m^7 x^{16}+28857216410 m^5 x^{18}+124530626231 m^3 x^{20}+29985521895 m x^{22}+60522 m^{10} x^{12}+78076350 m^8 x^{14}+10282782510 m^6 x^{16}+174273100210 m^4 x^{18}+292163767533 m^2 x^{20}+13749310575 x^{22}+462 m^{11} x^{10}+3471930 m^9 x^{12}+1613983140 m^7 x^{14}+93862508190 m^5 x^{16}+680615362515 m^3 x^{18}+360568238085 m x^{20}+61446 m^{10} x^{10}+114642990 m^8 x^{12}+22164925860 m^6 x^{14}+572017996770 m^4 x^{16}+1604842704135 m^2 x^{18}+165646455975 x^{20}+330 m^{11} x^8+3582810 m^9 x^{10}+2408820876 m^7 x^{12}+204865733820 m^5 x^{14}+2251106854425 m^3 x^{16}+1988025402825 m x^{18}+44550 m^{10} x^8+120367170 m^8 x^{10}+33609870756 m^6 x^{12}+1262375264700 m^4 x^{14}+5340787250535 m^2 x^{16}+915414625125 x^{18}+165 m^{11} x^6+2640990 m^9 x^8+2575140876 m^7 x^{10}+315347150580 m^5 x^{12}+5015196628530 m^3 x^{14}+6646727085075 m x^{16}+22605 m^{10} x^6+90358290 m^8 x^8+36597992508 m^6 x^{10}+1969992823260 m^4 x^{12}+11991258123570 m^2 x^{14}+3069331390125 x^{16}+55 m^{11} x^4+1362735 m^9 x^6+1971903780 m^7 x^8+349697552820 m^5 x^{10}+7921249136262 m^3 x^{12}+15011348834790 m x^{14}+7645 m^{10} x^4+47524455 m^8 x^6+28627538940 m^6 x^8+2222832699780 m^4 x^{10}+19130651800722 m^2 x^{12}+6957151150950 x^{14}+11 m^{11} x^2+468765 m^9 x^4+1059893010 m^7 x^6+279691771260 m^5 x^8+9079996141062 m^3 x^{10}+24133835554290 m x^{12}+1551 m^{10} x^2+16677375 m^8 x^4+15768085410 m^6 x^6+1818135330660 m^4 x^8+22226933020446 m^2 x^{10}+11238474936150 x^{12}+m^{11}+96745 m^9 x^2+380801190 m^7 x^4+158293212990 m^5 x^6+7587607623090 m^3 x^8+28336045738770 m x^{10}+143 m^{10}+3514005 m^8 x^2+5825106210 m^6 x^4+1059628145070 m^4 x^6+18930738943710 m^2 x^8+13281834015450 x^{10}+9075 m^9+82295598 m^7 x^2+60431072570 m^5 x^4+4558015784025 m^3 x^6+24503570194950 m x^8+336765 m^8+1298935638 m^6 x^2+420404849150 m^4 x^4+11703493287585 m^2 x^6+11595251918250 x^8+8103018 m^7+14014513810 m^5 x^2+1889780020755 m^3 x^4+15515657331075 m x^6+132426294 m^6+102468500970 m^4 x^2+5087634488145 m^2 x^4+7454090518875 x^6+1495875590 m^5+490955350391 m^3 x^2+7041864340665 m x^4+11641582810 m^4+1434440867211 m^2 x^2+3478575575475 x^4+60936676581 m^3+2192684754645 m x^2+203363952363 m^2+1159525191825 x^2+387182170935 m+316234143225\big) x\Big) \Big/ \big((1+m)(3+m)(5+m)(7+m)(9+m)(11+m)(13+m)(15+m)(17+m)(19+m)(21+m)(23+m)\big)$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int x^5 \left(x^2+1\right) \left(x^4+2 x^2+1\right)^5 dx$$

Optimal(type 1, 28 leaves, 4 steps):

$$\frac{\left(x^2+1\right)^{12}}{24}-\frac{\left(x^2+1\right)^{13}}{13}+\frac{\left(x^2+1\right)^{14}}{28}$$

Result(type 1, 61 leaves):

$$\frac{1}{28} x^{28}+\frac{11}{26} x^{26}+\frac{55}{24} x^{24}+\frac{15}{2} x^{22}+\frac{33}{2} x^{20}+\frac{77}{3} x^{18}+\frac{231}{8} x^{16}+\frac{165}{7} x^{14}+\frac{55}{4} x^{12}+\frac{11}{2} x^{10}+\frac{11}{8} x^8+\frac{1}{6} x^6$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int x \left(x^2+1\right) \left(x^4+2 x^2+1\right)^5 dx$$

Optimal(type 1, 9 leaves, 2 steps):

$$\frac{\left(x^2+1\right)^{12}}{24}$$

Result(type 1, 61 leaves):

$$\frac{1}{24} x^{24}+\frac{1}{2} x^{22}+\frac{11}{4} x^{20}+\frac{55}{6} x^{18}+\frac{165}{8} x^{16}+33 x^{14}+\frac{77}{2} x^{12}+33 x^{10}+\frac{165}{8} x^8+\frac{55}{6} x^6+\frac{11}{4} x^4+\frac{1}{2} x^2$$

Problem 26: Unable to integrate problem.

$$\int \frac{(f x)^m \left(e x^2+d\right)}{\left(b^2 x^4+2 a b x^2+a^2\right)^{3/2}} dx$$

Optimal(type 5, 126 leaves, 3 steps):

$$\frac{(-a e+b d) (f x)^{1+m}}{4 a b f \left(b x^2+a\right) \sqrt{\left(b x^2+a\right)^2}}+\frac{\left(b d (3-m)+a e (1+m)\right) (f x)^{1+m} \left(b x^2+a\right) \operatorname{hypergeom}\left(\left[2, \frac{1}{2}+\frac{m}{2}\right], \left[\frac{3}{2}+\frac{m}{2}\right], -\frac{b x^2}{a}\right)}{4 a^3 b f (1+m) \sqrt{\left(b x^2+a\right)^2}}$$

Result(type 8, 35 leaves):

$$\int \frac{(f x)^m \left(e x^2+d\right)}{\left(b^2 x^4+2 a b x^2+a^2\right)^{3/2}} dx$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int \frac{x^5 \left(B x^2+A\right)}{c x^4+b x^2+a} dx$$

Optimal(type 3, 121 leaves, 7 steps):

$$-\frac{(-Ac+bB)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{(-Abc-aBc+b^2B)\ln(cx^4+bx^2+a)}{4c^3}+\frac{(2aAc^2-Ab^2c-3abBc+b^3B)\operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{2c^3\sqrt{-4ac+b^2}}$$

Result(type 3, 260 leaves):

$$\frac{Bx^4}{4c}+\frac{Ax^2}{2c}-\frac{bBx^2}{2c^2}-\frac{\ln(cx^4+bx^2+a)Ab}{4c^2}-\frac{\ln(cx^4+bx^2+a)aB}{4c^2}+\frac{\ln(cx^4+bx^2+a)b^2B}{4c^3}-\frac{\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)aA}{c\sqrt{4ac-b^2}}$$
$$+\frac{3\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)abB}{2c^2\sqrt{4ac-b^2}}+\frac{\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)Ab^2}{2c^2\sqrt{4ac-b^2}}-\frac{\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)b^3B}{2c^3\sqrt{4ac-b^2}}$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int\frac{x^5(Bx^2+A)}{(cx^4+bx^2+a)^2}\,dx$$

Optimal(type 3, 137 leaves, 6 steps):

$$-\frac{x^2(a(-2Ac+bB)+(-Abc-2aBc+b^2B)x^2)}{2c(-4ac+b^2)(cx^4+bx^2+a)}+\frac{(4aAc^2-6abBc+b^3B)\operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{2c^2(-4ac+b^2)^{3/2}}+\frac{B\ln(cx^4+bx^2+a)}{4c^2}$$

Result(type 3, 541 leaves):

$$\frac{-\frac{(2aAc^2-Ab^2c-3abBc+b^3B)x^2}{c^2(4ac-b^2)}+\frac{a(Abc+2aBc-b^2B)}{c^2(4ac-b^2)}}{2(cx^4+bx^2+a)}+\frac{\ln((4ac-b^2)c(cx^4+bx^2+a))aB}{c(4ac-b^2)}$$
$$-\frac{\ln((4ac-b^2)c(cx^4+bx^2+a))b^2B}{4c^2(4ac-b^2)}+\frac{2\arctan\left(\frac{2c^2(4ac-b^2)x^2+(4ac-b^2)bc}{\sqrt{64a^3c^5-48a^2b^2c^4+12ab^4c^3-b^6c^2}}\right)aAc}{\sqrt{64a^3c^5-48a^2b^2c^4+12ab^4c^3-b^6c^2}}$$
$$-\frac{3\arctan\left(\frac{2c^2(4ac-b^2)x^2+(4ac-b^2)bc}{\sqrt{64a^3c^5-48a^2b^2c^4+12ab^4c^3-b^6c^2}}\right)abB}{\sqrt{64a^3c^5-48a^2b^2c^4+12ab^4c^3-b^6c^2}}+\frac{\arctan\left(\frac{2c^2(4ac-b^2)x^2+(4ac-b^2)bc}{\sqrt{64a^3c^5-48a^2b^2c^4+12ab^4c^3-b^6c^2}}\right)b^3B}{2\sqrt{64a^3c^5-48a^2b^2c^4+12ab^4c^3-b^6c^2}\,c}$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int\frac{Bx^2+A}{x^3(cx^4+bx^2+a)^2}\,dx$$

Optimal(type 3, 209 leaves, 8 steps):

$$\frac{6 a A c-2 A b^2+a b B}{2 a^2 \left(-4 a c+b^2\right) x^2}+\frac{-a b B+A \left(-2 a c+b^2\right)+(A b-2 a B) c x^2}{2 a \left(-4 a c+b^2\right) x^2 \left(c x^4+b x^2+a\right)}+\frac{\left(a b B \left(-6 a c+b^2\right)-2 A \left(6 a^2 c^2-6 a b^2 c+b^4\right)\right) \operatorname{arctanh}\left(\frac{2 c x^2+b}{\sqrt{-4 a c+b^2}}\right)}{2 a^3 \left(-4 a c+b^2\right)^{3/2}}$$
$$-\frac{(2 A b-a B) \ln(x)}{a^3}+\frac{(2 A b-a B) \ln\left(c x^4+b x^2+a\right)}{4 a^3}$$

Result(type 3, 990 leaves):

$$-\frac{c^2 x^2 A}{a \left(c x^4+b x^2+a\right) \left(4 a c-b^2\right)}+\frac{c x^2 A b^2}{2 a^2 \left(c x^4+b x^2+a\right) \left(4 a c-b^2\right)}-\frac{c x^2 b B}{2 a \left(c x^4+b x^2+a\right) \left(4 a c-b^2\right)}-\frac{3 A b c}{2 a \left(c x^4+b x^2+a\right) \left(4 a c-b^2\right)}$$
$$+\frac{A b^3}{2 a^2 \left(c x^4+b x^2+a\right) \left(4 a c-b^2\right)}+\frac{B c}{\left(c x^4+b x^2+a\right) \left(4 a c-b^2\right)}-\frac{B b^2}{2 a \left(c x^4+b x^2+a\right) \left(4 a c-b^2\right)}+\frac{2 c \ln\left(\left(4 a c-b^2\right) \left(c x^4+b x^2+a\right)\right) A b}{a^2 \left(4 a c-b^2\right)}$$
$$-\frac{\ln\left(\left(4 a c-b^2\right) \left(c x^4+b x^2+a\right)\right) A b^3}{2 a^3 \left(4 a c-b^2\right)}-\frac{c \ln\left(\left(4 a c-b^2\right) \left(c x^4+b x^2+a\right)\right) B}{a \left(4 a c-b^2\right)}+\frac{\ln\left(\left(4 a c-b^2\right) \left(c x^4+b x^2+a\right)\right) b^2 B}{4 a^2 \left(4 a c-b^2\right)}$$
$$-\frac{6 \arctan\left(\frac{2 \left(4 a c-b^2\right) c x^2+\left(4 a c-b^2\right) b}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}\right) A c^2}{a \sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}+\frac{6 \arctan\left(\frac{2 \left(4 a c-b^2\right) c x^2+\left(4 a c-b^2\right) b}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}\right) A b^2 c}{a^2 \sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}$$
$$-\frac{\arctan\left(\frac{2 \left(4 a c-b^2\right) c x^2+\left(4 a c-b^2\right) b}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}\right) A b^4}{a^3 \sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}-\frac{3 \arctan\left(\frac{2 \left(4 a c-b^2\right) c x^2+\left(4 a c-b^2\right) b}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}\right) B c b}{a \sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}$$
$$+\frac{\arctan\left(\frac{2 \left(4 a c-b^2\right) c x^2+\left(4 a c-b^2\right) b}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}\right) B b^3}{2 a^2 \sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}-\frac{A}{2 a^2 x^2}-\frac{2 \ln(x) A b}{a^3}+\frac{\ln(x) B}{a^2}$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int \frac{x^6 \left(B x^2+A\right)}{\left(c x^4+b x^2+a\right)^2} \, dx$$

Optimal(type 3, 377 leaves, 6 steps):

$$\frac{\left(-A b c-10 a B c+3 b^2 B\right) x}{2 c^2 \left(-4 a c+b^2\right)}-\frac{\left(-2 A c+b B\right) x^3}{2 c \left(-4 a c+b^2\right)}-\frac{x^5 \left(A b-2 a B-\left(-2 A c+b B\right) x^2\right)}{2 \left(-4 a c+b^2\right) \left(c x^4+b x^2+a\right)}$$
$$-\frac{\arctan\left(\frac{x \sqrt{2} \sqrt{c}}{\sqrt{b-\sqrt{-4 a c+b^2}}}\right) \left(3 b^3 B-A b^2 c-13 a b B c+6 a A c^2+\frac{-8 a A b c^2+A b^3 c-20 a^2 B c^2+19 a b^2 B c-3 b^4 B}{\sqrt{-4 a c+b^2}}\right) \sqrt{2}}{4 c^{5/2} \left(-4 a c+b^2\right) \sqrt{b-\sqrt{-4 a c+b^2}}}$$

$$-\frac{\arctan\left(\frac{x\sqrt{2}\,\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(3b^3B-Ab^2c-13abBc+6aAc^2+\frac{8aAbc^2-Ab^3c+20a^2Bc^2-19ab^2Bc+3b^4B}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4c^{5/2}\left(-4ac+b^2\right)\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type ?, 4262 leaves): Display of huge result suppressed!

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \frac{x^4\left(Bx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}\,dx$$

Optimal(type 3, 293 leaves, 5 steps):

$$-\frac{\left(-2Ac+bB\right)x}{2c\left(-4ac+b^2\right)}-\frac{x^3\left(Ab-2aB-\left(-2Ac+bB\right)x^2\right)}{2\left(-4ac+b^2\right)\left(cx^4+bx^2+a\right)}$$
$$+\frac{\arctan\left(\frac{x\sqrt{2}\,\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\left(b^2B+Abc-6aBc+\frac{-4aAc^2-Ab^2c+8abBc-b^3B}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4c^{3/2}\left(-4ac+b^2\right)\sqrt{b-\sqrt{-4ac+b^2}}}$$
$$+\frac{\arctan\left(\frac{x\sqrt{2}\,\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(b^2B+Abc-6aBc+\frac{4aAc^2+Ab^2c-8abBc+b^3B}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4c^{3/2}\left(-4ac+b^2\right)\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type ?, 4008 leaves): Display of huge result suppressed!

Problem 35: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2\left(Bx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}\,dx$$

Optimal(type 3, 234 leaves, 4 steps):

$$-\frac{x\left(Ab-2aB-\left(-2Ac+bB\right)x^2\right)}{2\left(-4ac+b^2\right)\left(cx^4+bx^2+a\right)}+\frac{\arctan\left(\frac{x\sqrt{2}\,\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\left(bB-2Ac+\frac{4Abc-4aBc-b^2B}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4\left(-4ac+b^2\right)\sqrt{c}\,\sqrt{b-\sqrt{-4ac+b^2}}}$$
$$+\frac{\arctan\left(\frac{x\sqrt{2}\,\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(bB-2Ac+\frac{-4Abc+4aBc+b^2B}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4\left(-4ac+b^2\right)\sqrt{c}\,\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type ?, 2994 leaves): Display of huge result suppressed!

Problem 36: Result more than twice size of optimal antiderivative.

$$\int \frac{B x^2 + A}{x^2 \left(c x^4 + b x^2 + a\right)^2}\, dx$$

Optimal(type 3, 337 leaves, 5 steps):

$$\frac{10 a A c - 3 A b^2 + a b B}{2 a^2 \left(-4 a c + b^2\right) x} + \frac{-a b B + A \left(-2 a c + b^2\right) + \left(A b - 2 a B\right) c x^2}{2 a \left(-4 a c + b^2\right) x \left(c x^4 + b x^2 + a\right)}$$
$$+ \frac{\arctan\left(\frac{x \sqrt{2}\, \sqrt{c}}{\sqrt{b - \sqrt{-4 a c + b^2}}}\right) \sqrt{c} \left(a B \left(b^2 - 12 a c + b \sqrt{-4 a c + b^2}\right) - A \left(3 b^3 - 16 a b c + 3 b^2 \sqrt{-4 a c + b^2} - 10 a c \sqrt{-4 a c + b^2}\right)\right) \sqrt{2}}{4 a^2 \left(-4 a c + b^2\right)^{3/2} \sqrt{b - \sqrt{-4 a c + b^2}}}$$
$$- \frac{\arctan\left(\frac{x \sqrt{2}\, \sqrt{c}}{\sqrt{b + \sqrt{-4 a c + b^2}}}\right) \sqrt{c} \left(3 A b^2 - a b B - 10 a A c + \frac{a B \left(-12 a c + b^2\right) - A \left(-16 a b c + 3 b^3\right)}{\sqrt{-4 a c + b^2}}\right) \sqrt{2}}{4 a^2 \left(-4 a c + b^2\right) \sqrt{b + \sqrt{-4 a c + b^2}}}$$

Result(type ?, 3751 leaves): Display of huge result suppressed!

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{x^{11} \left(B x^2 + A\right)}{\left(c x^4 + b x^2 + a\right)^3}\, dx$$

Optimal(type 3, 351 leaves, 8 steps):

$$\frac{\left(7 a A b c^2 - A b^3 c + 30 a^2 B c^2 - 21 a b^2 B c + 3 b^4 B\right) x^2}{2 c^3 \left(-4 a c + b^2\right)^2} - \frac{x^8 \left(a \left(-2 A c + b B\right) + \left(-A b c - 2 a B c + b^2 B\right) x^2\right)}{4 c \left(-4 a c + b^2\right) \left(c x^4 + b x^2 + a\right)^2}$$
$$- \frac{x^4 \left(a \left(16 a A c^2 - A b^2 c - 18 a b B c + 3 b^3 B\right) + \left(10 a A b c^2 - A b^3 c + 20 a^2 B c^2 - 20 a b^2 B c + 3 b^4 B\right) x^2\right)}{4 c^2 \left(-4 a c + b^2\right)^2 \left(c x^4 + b x^2 + a\right)}$$
$$- \frac{\left(-30 a^2 A b c^3 + 10 a A b^3 c^2 - A b^5 c - 60 a^3 B c^3 + 90 a^2 b^2 B c^2 - 30 a b^4 B c + 3 b^6 B\right) \operatorname{arctanh}\left(\frac{2 c x^2 + b}{\sqrt{-4 a c + b^2}}\right)}{2 c^4 \left(-4 a c + b^2\right)^{5/2}} - \frac{\left(-A c + 3 b B\right) \ln\left(c x^4 + b x^2 + a\right)}{4 c^4}$$

Result(type ?, 2915 leaves): Display of huge result suppressed!

Problem 38: Result more than twice size of optimal antiderivative.

$$\int \frac{x^7 \left(B x^2 + A\right)}{\left(c x^4 + b x^2 + a\right)^3}\, dx$$

Optimal(type 3, 138 leaves, 5 steps):

$$-\frac{x^6\left(Ab-2aB-(-2Ac+bB)x^2\right)}{4\left(-4ac+b^2\right)\left(cx^4+bx^2+a\right)^2}+\frac{3\left(Ab-2aB\right)x^2\left(bx^2+2a\right)}{4\left(-4ac+b^2\right)^2\left(cx^4+bx^2+a\right)}+\frac{3a\left(Ab-2aB\right)\operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{\left(-4ac+b^2\right)^{5/2}}$$

Result(type 3, 397 leaves):

$$\frac{1}{2\left(cx^4+bx^2+a\right)^2}\left(-\frac{\left(3aAbc^2+10a^2Bc^2-8ab^2Bc+b^4B\right)x^6}{c\left(16a^2c^2-8ab^2c+b^4\right)}-\frac{\left(16Aa^2c^3+Aab^2c^2+Ab^4c-2Ba^2bc^2-8Bab^3c+Bb^5\right)x^4}{2\left(16a^2c^2-8ab^2c+b^4\right)c^2}\right.$$
$$\left.-\frac{a\left(5aAbc^2+Ab^3c+6a^2Bc^2-10ab^2Bc+b^4B\right)x^2}{c^2\left(16a^2c^2-8ab^2c+b^4\right)}-\frac{a^2\left(8aAc^2+Ab^2c-10abBc+b^3B\right)}{2c^2\left(16a^2c^2-8ab^2c+b^4\right)}\right)-\frac{3a\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)Ab}{\left(16a^2c^2-8ab^2c+b^4\right)\sqrt{4ac-b^2}}$$
$$+\frac{6a^2\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)B}{\left(16a^2c^2-8ab^2c+b^4\right)\sqrt{4ac-b^2}}$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int\frac{Bx^2+A}{x^3\left(cx^4+bx^2+a\right)^3}\,dx$$

Optimal(type 3, 349 leaves, 9 steps):

$$\frac{abB\left(-7ac+b^2\right)-3A\left(10a^2c^2-7ab^2c+b^4\right)}{2a^3\left(-4ac+b^2\right)^2x^2}+\frac{-abB+A\left(-2ac+b^2\right)+\left(Ab-2aB\right)cx^2}{4a\left(-4ac+b^2\right)x^2\left(cx^4+bx^2+a\right)^2}$$
$$+\frac{-abB\left(-10ac+b^2\right)+A\left(20a^2c^2-20ab^2c+3b^4\right)-c\left(aB\left(-16ac+b^2\right)-3A\left(-6abc+b^3\right)\right)x^2}{4a^2\left(-4ac+b^2\right)^2x^2\left(cx^4+bx^2+a\right)}$$
$$+\frac{\left(abB\left(30a^2c^2-10ab^2c+b^4\right)-3A\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\right)\operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{2a^4\left(-4ac+b^2\right)^{5/2}}-\frac{\left(3Ab-aB\right)\ln(x)}{a^4}$$
$$+\frac{\left(3Ab-aB\right)\ln\left(cx^4+bx^2+a\right)}{4a^4}$$

Result(type ?, 2723 leaves): Display of huge result suppressed!

Problem 40: Result more than twice size of optimal antiderivative.

$$\int\frac{x^6\left(Bx^2+A\right)}{\left(cx^4+bx^2+a\right)^3}\,dx$$

Optimal(type 3, 415 leaves, 6 steps):

$$-\frac{(-12Abc+20aBc+b^2B)x}{8c(-4ac+b^2)^2}-\frac{x^5(Ab-2aB-(-2Ac+bB)x^2)}{4(-4ac+b^2)(cx^4+bx^2+a)^2}-\frac{x^3(5Ab^2-12abB+4aAc-(-12Abc+20aBc+b^2B)x^2)}{8(-4ac+b^2)^2(cx^4+bx^2+a)}$$
$$+\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\left(b^3B+3Ab^2c-16abBc+12aAc^2+\frac{-36aAbc^2-3Ab^3c+40a^2Bc^2+18ab^2Bc-b^4B}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{16c^{3/2}(-4ac+b^2)^2\sqrt{b-\sqrt{-4ac+b^2}}}$$
$$+\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(b^3B+3Ab^2c-16abBc+12aAc^2+\frac{36aAbc^2+3Ab^3c-40a^2Bc^2-18ab^2Bc+b^4B}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{16c^{3/2}(-4ac+b^2)^2\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type ?, 9167 leaves): Display of huge result suppressed!

Problem 53: Unable to integrate problem.

$$\int\sqrt{fx}\,(ex^2+d)\sqrt{cx^4+bx^2+a}\,dx$$

Optimal(type 6, 245 leaves, 6 steps):

$$\frac{2d(fx)^{3/2}\mathit{AppellF1}\left(\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2x^2c}{b-\sqrt{-4ac+b^2}},-\frac{2x^2c}{b+\sqrt{-4ac+b^2}}\right)\sqrt{cx^4+bx^2+a}}{3f\sqrt{1+\frac{2x^2c}{b-\sqrt{-4ac+b^2}}}\sqrt{1+\frac{2x^2c}{b+\sqrt{-4ac+b^2}}}}$$
$$+\frac{2e(fx)^{7/2}\mathit{AppellF1}\left(\frac{7}{4},-\frac{1}{2},-\frac{1}{2},\frac{11}{4},-\frac{2x^2c}{b-\sqrt{-4ac+b^2}},-\frac{2x^2c}{b+\sqrt{-4ac+b^2}}\right)\sqrt{cx^4+bx^2+a}}{7f^3\sqrt{1+\frac{2x^2c}{b-\sqrt{-4ac+b^2}}}\sqrt{1+\frac{2x^2c}{b+\sqrt{-4ac+b^2}}}}$$

Result(type 8, 144 leaves):

$$\frac{2x^2(7cex^2+2be+11cd)\sqrt{cx^4+bx^2+a}\,f}{77c\sqrt{fx}}+\frac{\left(\int-\frac{2x(-14acex^2+5b^2ex^2-11bcdx^2+3abe-22adc)}{77c\sqrt{(cx^4+bx^2+a)fx}}\,dx\right)f\sqrt{(cx^4+bx^2+a)fx}}{\sqrt{fx}\sqrt{cx^4+bx^2+a}}$$

Problem 54: Unable to integrate problem.

$$\int\frac{(ex^2+d)\sqrt{cx^4+bx^2+a}}{(fx)^{3/2}}\,dx$$

Optimal(type 6, 245 leaves, 6 steps):

$$\frac{2 e (fx)^{3/2} \operatorname{AppellF1}\left(\frac{3}{4}, -\frac{1}{2}, -\frac{1}{2}, \frac{7}{4}, -\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}, -\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}\right) \sqrt{c x^4+b x^2+a}}{3 f^3 \sqrt{1+\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}} \sqrt{1+\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}}}$$
$$-\frac{2 d \operatorname{AppellF1}\left(-\frac{1}{4}, -\frac{1}{2}, -\frac{1}{2}, \frac{3}{4}, -\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}, -\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}\right) \sqrt{c x^4+b x^2+a}}{f \sqrt{fx} \sqrt{1+\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}} \sqrt{1+\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}}}$$

Result(type 8, 119 leaves):

$$-\frac{2 \sqrt{c x^4+b x^2+a}\,(-e x^2+7 d)}{7 f \sqrt{fx}} + \frac{\left(\int \frac{2 x (b e x^2+14 c d x^2+2 a e+7 b d)}{7 \sqrt{(c x^4+b x^2+a) f x}}\, dx\right) \sqrt{(c x^4+b x^2+a) f x}}{f \sqrt{fx} \sqrt{c x^4+b x^2+a}}$$

Problem 55: Unable to integrate problem.

$$\int (fx)^{3/2} (e x^2+d) (c x^4+b x^2+a)^{3/2}\, dx$$

Optimal(type 6, 247 leaves, 6 steps):

$$\frac{2 a d (fx)^{5/2} \operatorname{AppellF1}\left(\frac{5}{4}, -\frac{3}{2}, -\frac{3}{2}, \frac{9}{4}, -\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}, -\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}\right) \sqrt{c x^4+b x^2+a}}{5 f \sqrt{1+\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}} \sqrt{1+\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}}}$$
$$+\frac{2 a e (fx)^{9/2} \operatorname{AppellF1}\left(\frac{9}{4}, -\frac{3}{2}, -\frac{3}{2}, \frac{13}{4}, -\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}, -\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}\right) \sqrt{c x^4+b x^2+a}}{9 f^3 \sqrt{1+\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}} \sqrt{1+\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}}}$$

Result(type 8, 348 leaves):

$$\frac{1}{69615 c^3 \sqrt{fx}} \Big(2 \big(3315 e x^8 c^4 + 4485 b c^3 e x^6 + 4095 c^4 d x^6 + 6375 a c^3 e x^4 + 180 b^2 c^2 e x^4 + 5985 b c^3 d x^4 + 1200 a b c^2 e x^2 + 9555 a c^3 d x^2 - 220 b^3 c e x^2$$
$$+ 420 b^2 c^2 d x^2 + 2448 a^2 e c^2 - 2004 a b^2 c e + 3696 a b d c^2 + 308 b^4 e - 588 b^3 c d\big) \sqrt{c x^4+b x^2+a}\, x f^2\Big) + \frac{1}{\sqrt{fx} \sqrt{c x^4+b x^2+a}} \Bigg(\Bigg(\int$$

$$-\frac{1}{69615 c^3 \sqrt{(c x^4 + b x^2 + a) f x}} \Big(4 \big(3336 a^2 b c^2 e x^2 - 5460 a^2 c^3 d x^2 - 1778 a b^3 c e x^2 + 3297 a b^2 c^2 d x^2 + 231 b^5 e x^2 - 441 b^4 c d x^2 + 612 a^3 c^2 e$$
$$- 501 a^2 b^2 c e + 924 a^2 b c^2 d + 77 a b^4 e - 147 a b^3 c d\big)\big) dx\Big) f^2 \sqrt{(c x^4 + b x^2 + a) f x}\Big)$$

Problem 56: Unable to integrate problem.

$$\int \frac{\sqrt{f x}\,(e x^2 + d)}{(c x^4 + b x^2 + a)^{3/2}}\, dx$$

Optimal(type 6, 251 leaves, 6 steps):

$$\frac{2 d (f x)^{3/2} \mathit{AppellF1}\left(\frac{3}{4}, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, -\frac{2 x^2 c}{b - \sqrt{-4 a c + b^2}}, -\frac{2 x^2 c}{b + \sqrt{-4 a c + b^2}}\right) \sqrt{1 + \frac{2 x^2 c}{b - \sqrt{-4 a c + b^2}}} \sqrt{1 + \frac{2 x^2 c}{b + \sqrt{-4 a c + b^2}}}}{3 a f \sqrt{c x^4 + b x^2 + a}}$$
$$+ \frac{2 e (f x)^{7/2} \mathit{AppellF1}\left(\frac{7}{4}, \frac{3}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 x^2 c}{b - \sqrt{-4 a c + b^2}}, -\frac{2 x^2 c}{b + \sqrt{-4 a c + b^2}}\right) \sqrt{1 + \frac{2 x^2 c}{b - \sqrt{-4 a c + b^2}}} \sqrt{1 + \frac{2 x^2 c}{b + \sqrt{-4 a c + b^2}}}}{7 a f^3 \sqrt{c x^4 + b x^2 + a}}$$

Result(type 8, 29 leaves):

$$\int \frac{\sqrt{f x}\,(e x^2 + d)}{(c x^4 + b x^2 + a)^{3/2}}\, dx$$

Problem 57: Result more than twice size of optimal antiderivative.

$$\int (f x)^m (e x^2 + d) (c x^4 + b x^2 + a)^3\, dx$$

Optimal(type 3, 243 leaves, 2 steps):

$$\frac{a^3 d (f x)^{1+m}}{f (1+m)} + \frac{a^2 (a e + 3 b d) (f x)^{3+m}}{f^3 (3+m)} + \frac{3 a \left(a b e + a d c + b^2 d\right) (f x)^{5+m}}{f^5 (5+m)} + \frac{\left(3 a^2 c e + 3 a b^2 e + 6 a b c d + b^3 d\right) (f x)^{7+m}}{f^7 (7+m)}$$
$$+ \frac{\left(6 a b c e + 3 a c^2 d + b^3 e + 3 b^2 c d\right) (f x)^{9+m}}{f^9 (9+m)} + \frac{3 c \left(a c e + b^2 e + b c d\right) (f x)^{11+m}}{f^{11} (11+m)} + \frac{c^2 (3 b e + c d) (f x)^{13+m}}{f^{13} (13+m)} + \frac{c^3 e (f x)^{15+m}}{f^{15} (15+m)}$$

Result(type 3, 1934 leaves):

$$\frac{1}{(1+m)(3+m)(5+m)(7+m)(9+m)(11+m)(13+m)(15+m)} \big(x \big(c^3 e m^7 x^{14} + 49 c^3 e m^6 x^{14} + 3 b c^2 e m^7 x^{12} + c^3 d m^7 x^{12} + 973 c^3 e m^5 x^{14}$$
$$+ 153 b c^2 e m^6 x^{12} + 51 c^3 d m^6 x^{12} + 10045 c^3 e m^4 x^{14} + 3 a c^2 e m^7 x^{10} + 3 b^2 c e m^7 x^{10} + 3 b c^2 d m^7 x^{10} + 3135 b c^2 e m^5 x^{12} + 1045 c^3 d m^5 x^{12}$$
$$+ 57379 c^3 e m^3 x^{14} + 159 a c^2 e m^6 x^{10} + 159 b^2 c e m^6 x^{10} + 159 b c^2 d m^6 x^{10} + 33165 b c^2 e m^4 x^{12} + 11055 c^3 d m^4 x^{12} + 177331 c^3 e m^2 x^{14} + 6 a b c e m^7 x^8$$
$$+ 3 a c^2 d m^7 x^8 + 3375 a c^2 e m^5 x^{10} + b^3 e m^7 x^8 + 3 b^2 c d m^7 x^8 + 3375 b^2 c e m^5 x^{10} + 3375 b c^2 d m^5 x^{10} + 193017 b c^2 e m^3 x^{12} + 64339 c^3 d m^3 x^{12}$$

$$+264207\,c^3 e m x^{14}+330\,a b c e m^6 x^8+165\,a c^2 d m^6 x^8+36795\,a c^2 e m^4 x^{10}+55\,b^3 e m^6 x^8+165\,b^2 c d m^6 x^8+36795\,b^2 c e m^4 x^{10}+36795\,b c^2 d m^4 x^{10}$$
$$+604827\,b c^2 e m^2 x^{12}+201609\,c^3 d m^2 x^{12}+135135\,e c^3 x^{14}+3\,a^2 c e m^7 x^6+3\,a b^2 e m^7 x^6+6\,a b c d m^7 x^6+7278\,a b c e m^5 x^8+3639\,a c^2 d m^5 x^8$$
$$+219417\,a c^2 e m^3 x^{10}+b^3 d m^7 x^6+1213\,b^3 e m^5 x^8+3639\,b^2 c d m^5 x^8+219417\,b^2 c e m^3 x^{10}+219417\,b c^2 d m^3 x^{10}+909765\,b c^2 e m x^{12}+303255\,c^3 d m x^{12}$$
$$+171\,a^2 c e m^6 x^6+171\,a b^2 e m^6 x^6+342\,a b c d m^6 x^6+82338\,a b c e m^4 x^8+41169\,a c^2 d m^4 x^8+700461\,a c^2 e m^2 x^{10}+57\,b^3 d m^6 x^6+13723\,b^3 e m^4 x^8$$
$$+41169\,b^2 c d m^4 x^8+700461\,b^2 c e m^2 x^{10}+700461\,b c^2 d m^2 x^{10}+467775\,b c^2 e x^{12}+155925\,c^3 d x^{12}+3\,a^2 b e m^7 x^4+3\,a^2 c d m^7 x^4+3927\,a^2 c e m^5 x^6$$
$$+3\,a b^2 d m^7 x^4+3927\,a b^2 e m^5 x^6+7854\,a b c d m^5 x^6+507282\,a b c e m^3 x^8+253641\,a c^2 d m^3 x^8+1067445\,a c^2 e m x^{10}+1309\,b^3 d m^5 x^6+84547\,b^3 e m^3 x^8$$
$$+253641\,b^2 c d m^3 x^8+1067445\,b^2 c e m x^{10}+1067445\,b c^2 d m x^{10}+177\,a^2 b e m^6 x^4+177\,a^2 c d m^6 x^4+46431\,a^2 c e m^4 x^6+177\,a b^2 d m^6 x^4$$
$$+46431\,a b^2 e m^4 x^6+92862\,a b c d m^4 x^6+1662558\,a b c e m^2 x^8+831279\,a c^2 d m^2 x^8+552825\,a c^2 e x^{10}+15477\,b^3 d m^4 x^6+277093\,b^3 e m^2 x^8$$
$$+831279\,b^2 c d m^2 x^8+552825\,b^2 c e x^{10}+552825\,b c^2 d x^{10}+a^3 e m^7 x^2+3\,a^2 b d m^7 x^2+4239\,a^2 b e m^5 x^4+4239\,a^2 c d m^5 x^4+299145\,a^2 c e m^3 x^6$$
$$+4239\,a b^2 d m^5 x^4+299145\,a b^2 e m^3 x^6+598290\,a b c d m^3 x^6+2582010\,a b c e m x^8+1291005\,a c^2 d m x^8+99715\,b^3 d m^3 x^6+430335\,b^3 e m x^8$$
$$+1291005\,b^2 c d m x^8+61\,a^3 e m^6 x^2+183\,a^2 b d m^6 x^2+52725\,a^2 b e m^4 x^4+52725\,a^2 c d m^4 x^4+1020033\,a^2 c e m^2 x^6+52725\,a b^2 d m^4 x^4$$
$$+1020033\,a b^2 e m^2 x^6+2040066\,a b c d m^2 x^6+1351350\,a b c e x^8+675675\,a c^2 d x^8+340011\,b^3 d m^2 x^6+225225\,b^3 e x^8+675675\,b^2 c d x^8+a^3 d m^7$$
$$+1525\,a^3 e m^5 x^2+4575\,a^2 b d m^5 x^2+360537\,a^2 b e m^3 x^4+360537\,a^2 c d m^3 x^4+1632285\,a^2 c e m x^6+360537\,a b^2 d m^3 x^4+1632285\,a b^2 e m x^6$$
$$+3264570\,a b c d m x^6+544095\,b^3 d m x^6+63\,a^3 d m^6+20065\,a^3 e m^4 x^2+60195\,a^2 b d m^4 x^2+1311363\,a^2 b e m^2 x^4+1311363\,a^2 c d m^2 x^4+868725\,a^2 c e x^6$$
$$+1311363\,a b^2 d m^2 x^4+868725\,a b^2 e x^6+1737450\,a b c d x^6+289575\,b^3 d x^6+1645\,a^3 d m^5+147859\,a^3 e m^3 x^2+443577\,a^2 b d m^3 x^2+2215701\,a^2 b e m x^4$$
$$+2215701\,a^2 c d m x^4+2215701\,a b^2 d m x^4+22995\,a^3 d m^4+594439\,a^3 e m^2 x^2+1783317\,a^2 b d m^2 x^2+1216215\,a^2 b e x^4+1216215\,a^2 c d x^4$$
$$+1216215\,a b^2 d x^4+185059\,a^3 d m^3+1140855\,a^3 e m x^2+3422565\,a^2 b d m x^2+852957\,a^3 d m^2+675675\,a^3 e x^2+2027025\,a^2 b d x^2+2071215\,a^3 d m$$
$$+2027025\,d a^3\Big)\,(f x)^m\Big)$$

Problem 58: Result more than twice size of optimal antiderivative.

$$\int (f x)^m \left(e x^2+d\right)\left(c x^4+b x^2+a\right)^2 dx$$

Optimal(type 3, 155 leaves, 2 steps):

$$\frac{a^2 d (f x)^{1+m}}{f (1+m)}+\frac{a (a e+2 b d) (f x)^{3+m}}{f^3 (3+m)}+\frac{\left(2 a b e+2 a d c+b^2 d\right) (f x)^{5+m}}{f^5 (5+m)}+\frac{\left(2 a c e+b^2 e+2 b c d\right) (f x)^{7+m}}{f^7 (7+m)}+\frac{c (2 b e+c d) (f x)^{9+m}}{f^9 (9+m)}$$
$$+\frac{c^2 e (f x)^{11+m}}{f^{11} (11+m)}$$

Result(type 3, 782 leaves):

$$\frac{1}{(11+m)(9+m)(7+m)(5+m)(3+m)(1+m)}\Big(x\,\big(c^2 e m^5 x^{10}+25\,c^2 e m^4 x^{10}+2\,b c e m^5 x^8+c^2 d m^5 x^8+230\,c^2 e m^3 x^{10}+54\,b c e m^4 x^8+27\,c^2 d m^4 x^8$$
$$+950\,c^2 e m^2 x^{10}+2\,a c e m^5 x^6+b^2 e m^5 x^6+2\,b c d m^5 x^6+524\,b c e m^3 x^8+262\,c^2 d m^3 x^8+1689\,c^2 e m x^{10}+58\,a c e m^4 x^6+29\,b^2 e m^4 x^6+58\,b c d m^4 x^6$$
$$+2244\,b c e m^2 x^8+1122\,c^2 d m^2 x^8+945\,c^2 e x^{10}+2\,a b e m^5 x^4+2\,a c d m^5 x^4+604\,a c e m^3 x^6+b^2 d m^5 x^4+302\,b^2 e m^3 x^6+604\,b c d m^3 x^6+4082\,b c e m x^8$$
$$+2041\,c^2 d m x^8+62\,a b e m^4 x^4+62\,a c d m^4 x^4+2732\,a c e m^2 x^6+31\,b^2 d m^4 x^4+1366\,b^2 e m^2 x^6+2732\,b c d m^2 x^6+2310\,b c e x^8+1155\,c^2 d x^8+a^2 e m^5 x^2$$
$$+2\,a b d m^5 x^2+700\,a b e m^3 x^4+700\,a c d m^3 x^4+5154\,a c e m x^6+350\,b^2 d m^3 x^4+2577\,b^2 e m x^6+5154\,b c d m x^6+33\,a^2 e m^4 x^2+66\,a b d m^4 x^2$$
$$+3460\,a b e m^2 x^4+3460\,a c d m^2 x^4+2970\,a c e x^6+1730\,b^2 d m^2 x^4+1485\,b^2 e x^6+2970\,b c d x^6+a^2 d m^5+406\,a^2 e m^3 x^2+812\,a b d m^3 x^2$$

$+ 6978\,a\,b\,e\,m\,x^4 + 6978\,a\,c\,d\,m\,x^4 + 3489\,b^2\,d\,m\,x^4 + 35\,a^2\,d\,m^4 + 2262\,a^2\,e\,m^2\,x^2 + 4524\,a\,b\,d\,m^2\,x^2 + 4158\,a\,b\,e\,x^4 + 4158\,a\,c\,d\,x^4 + 2079\,b^2\,d\,x^4$
$+ 470\,a^2\,d\,m^3 + 5353\,a^2\,e\,m\,x^2 + 10706\,a\,b\,d\,m\,x^2 + 3010\,a^2\,d\,m^2 + 3465\,a^2\,x^2\,e + 6930\,a\,b\,d\,x^2 + 9129\,a^2\,d\,m + 10395\,d\,a^2\big)\,(f x)^m\big)$

Problem 59: Unable to integrate problem.

$$\int (f x)^m \left(e x^2 + d\right) \left(c x^4 + b x^2 + a\right)^{3/2} \mathrm{d}x$$

Optimal(type 6, 275 leaves, 6 steps):

$$\frac{a\,d\,(f x)^{1+m}\,\mathit{AppellF1}\left(\frac{1}{2}+\frac{m}{2}, -\frac{3}{2}, -\frac{3}{2}, \frac{3}{2}+\frac{m}{2}, -\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}, -\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}\right)\sqrt{c x^4+b x^2+a}}{f\,(1+m)\sqrt{1+\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}}\,\sqrt{1+\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}}}$$
$$+\frac{a\,e\,(f x)^{3+m}\,\mathit{AppellF1}\left(\frac{3}{2}+\frac{m}{2}, -\frac{3}{2}, -\frac{3}{2}, \frac{5}{2}+\frac{m}{2}, -\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}, -\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}\right)\sqrt{c x^4+b x^2+a}}{f^3\,(3+m)\sqrt{1+\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}}\,\sqrt{1+\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}}}$$

Result(type 8, 29 leaves):

$$\int (f x)^m \left(e x^2 + d\right) \left(c x^4 + b x^2 + a\right)^{3/2} \mathrm{d}x$$

Problem 60: Unable to integrate problem.

$$\int \frac{(f x)^m \left(e x^2 + d\right)}{\left(c x^4 + b x^2 + a\right)^{3/2}} \mathrm{d}x$$

Optimal(type 6, 279 leaves, 6 steps):

$$\frac{d\,(f x)^{1+m}\,\mathit{AppellF1}\left(\frac{1}{2}+\frac{m}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}+\frac{m}{2}, -\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}, -\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}\right)\sqrt{1+\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}}\,\sqrt{1+\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}}}{a\,f\,(1+m)\sqrt{c x^4+b x^2+a}}$$
$$+\frac{e\,(f x)^{3+m}\,\mathit{AppellF1}\left(\frac{3}{2}+\frac{m}{2}, \frac{3}{2}, \frac{3}{2}, \frac{5}{2}+\frac{m}{2}, -\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}, -\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}\right)\sqrt{1+\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}}\,\sqrt{1+\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}}}{a\,f^3\,(3+m)\sqrt{c x^4+b x^2+a}}$$

Result(type 8, 29 leaves):

$$\int \frac{(f x)^m \left(e x^2 + d\right)}{\left(c x^4 + b x^2 + a\right)^{3/2}} \mathrm{d}x$$

Problem 72: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{(-x^2+1)\sqrt{x^4-1}}\, dx$$

Optimal(type 4, 45 leaves, 4 steps):

$$\frac{x(x^2+1)}{2\sqrt{x^4-1}} - \frac{\text{EllipticE}(x, I)\sqrt{-x^2+1}\sqrt{x^2+1}}{2\sqrt{x^4-1}}$$

Result(type 4, 133 leaves):

$$\frac{I\sqrt{x^2+1}\sqrt{-x^2+1}\,\text{EllipticF}(Ix, I)}{2\sqrt{x^4-1}} + \frac{x^3+x^2+x+1}{4\sqrt{(x-1)(x^3+x^2+x+1)}} + \frac{I\sqrt{x^2+1}\sqrt{-x^2+1}\,(\text{EllipticF}(Ix, I) - \text{EllipticE}(Ix, I))}{2\sqrt{x^4-1}} + \frac{x^3-x^2+x-1}{4\sqrt{(x+1)(x^3-x^2+x-1)}}$$

Problem 79: Result more than twice size of optimal antiderivative.

$$\int \frac{x^9}{(ex^2+d)(cx^4+bx^2+a)}\, dx$$

Optimal(type 3, 216 leaves, 7 steps):

$$-\frac{(be+cd)x^2}{2c^2e^2} + \frac{x^4}{4ce} + \frac{d^4\ln(ex^2+d)}{2e^3(ae^2-bde+cd^2)} - \frac{(a^2ce-ab^2e-2abcd+b^3d)\ln(cx^4+bx^2+a)}{4c^3(ae^2-bde+cd^2)} - \frac{(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d)\,\text{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{2c^3(ae^2-bde+cd^2)\sqrt{-4ac+b^2}}$$

Result(type 3, 537 leaves):

$$\frac{x^4}{4ce} - \frac{x^2b}{2c^2e} - \frac{x^2d}{2ce^2} - \frac{\ln(cx^4+bx^2+a)a^2e}{4(ae^2-bde+cd^2)c^2} + \frac{\ln(cx^4+bx^2+a)ab^2e}{4(ae^2-bde+cd^2)c^3} + \frac{\ln(cx^4+bx^2+a)abd}{2(ae^2-bde+cd^2)c^2} - \frac{\ln(cx^4+bx^2+a)b^3d}{4(ae^2-bde+cd^2)c^3} + \frac{3\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)a^2be}{2(ae^2-bde+cd^2)c^2\sqrt{4ac-b^2}} + \frac{\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)a^2d}{(ae^2-bde+cd^2)c\sqrt{4ac-b^2}} - \frac{2\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)ab^2d}{(ae^2-bde+cd^2)c^2\sqrt{4ac-b^2}} - \frac{\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)b^3ae}{2(ae^2-bde+cd^2)c^3\sqrt{4ac-b^2}} + \frac{\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)b^4d}{2(ae^2-bde+cd^2)c^3\sqrt{4ac-b^2}} + \frac{d^4\ln(ex^2+d)}{2e^3(ae^2-bde+cd^2)}$$

Problem 81: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^5 \left(e x^2+d\right) \left(c x^4+b x^2+a\right)}\, dx$$

Optimal(type 3, 254 leaves, 7 steps):

$$-\frac{1}{4 a d x^4}+\frac{a e+b d}{2 a^2 d^2 x^2}+\frac{\left(b^2 d^2+a b d e-a \left(-a e^2+c d^2\right)\right) \ln(x)}{a^3 d^3}-\frac{e^4 \ln\left(e x^2+d\right)}{2 d^3 \left(a e^2-b d e+c d^2\right)}-\frac{\left(2 a b c e-a c^2 d-b^3 e+b^2 c d\right) \ln\left(c x^4+b x^2+a\right)}{4 a^3 \left(a e^2-b d e+c d^2\right)}$$
$$+\frac{\left(-2 a^2 e c^2+4 a b^2 c e-3 a b d c^2-b^4 e+b^3 c d\right) \operatorname{arctanh}\left(\frac{2 c x^2+b}{\sqrt{-4 a c+b^2}}\right)}{2 a^3 \left(a e^2-b d e+c d^2\right) \sqrt{-4 a c+b^2}}$$

Result(type 3, 583 leaves):

$$-\frac{c \ln\left(c x^4+b x^2+a\right) b e}{2 a^2 \left(a e^2-b d e+c d^2\right)}+\frac{c^2 \ln\left(c x^4+b x^2+a\right) d}{4 a^2 \left(a e^2-b d e+c d^2\right)}+\frac{\ln\left(c x^4+b x^2+a\right) b^3 e}{4 a^3 \left(a e^2-b d e+c d^2\right)}-\frac{c \ln\left(c x^4+b x^2+a\right) b^2 d}{4 a^3 \left(a e^2-b d e+c d^2\right)}+\frac{\arctan\left(\frac{2 c x^2+b}{\sqrt{4 a c-b^2}}\right) e c^2}{a \left(a e^2-b d e+c d^2\right) \sqrt{4 a c-b^2}}$$
$$-\frac{2 \arctan\left(\frac{2 c x^2+b}{\sqrt{4 a c-b^2}}\right) b^2 c e}{a^2 \left(a e^2-b d e+c d^2\right) \sqrt{4 a c-b^2}}+\frac{3 \arctan\left(\frac{2 c x^2+b}{\sqrt{4 a c-b^2}}\right) b d c^2}{2 a^2 \left(a e^2-b d e+c d^2\right) \sqrt{4 a c-b^2}}+\frac{\arctan\left(\frac{2 c x^2+b}{\sqrt{4 a c-b^2}}\right) b^4 e}{2 a^3 \left(a e^2-b d e+c d^2\right) \sqrt{4 a c-b^2}}$$
$$-\frac{\arctan\left(\frac{2 c x^2+b}{\sqrt{4 a c-b^2}}\right) b^3 c d}{2 a^3 \left(a e^2-b d e+c d^2\right) \sqrt{4 a c-b^2}}-\frac{1}{4 a d x^4}+\frac{e}{2 d^2 a x^2}+\frac{b}{2 d a^2 x^2}+\frac{\ln(x) e^2}{d^3 a}+\frac{\ln(x) b e}{d^2 a^2}-\frac{\ln(x) c}{d a^2}+\frac{\ln(x) b^2}{d a^3}-\frac{e^4 \ln\left(e x^2+d\right)}{2 d^3 \left(a e^2-b d e+c d^2\right)}$$

Problem 82: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4 \left(e x^2+d\right) \left(c x^4+b x^2+a\right)}\, dx$$

Optimal(type 3, 303 leaves, 6 steps):

$$-\frac{1}{3 a d x^3}+\frac{a e+b d}{a^2 d^2 x}+\frac{e^{7/2} \arctan\left(\frac{x \sqrt{e}}{\sqrt{d}}\right)}{d^{5/2} \left(a e^2-b d e+c d^2\right)}+\frac{\arctan\left(\frac{x \sqrt{2}\, \sqrt{c}}{\sqrt{b-\sqrt{-4 a c+b^2}}}\right) \sqrt{c}\, \left(b c d-b^2 e+a c e+\frac{3 a b c e-2 a c^2 d-b^3 e+b^2 c d}{\sqrt{-4 a c+b^2}}\right) \sqrt{2}}{2 a^2 \left(a e^2-b d e+c d^2\right) \sqrt{b-\sqrt{-4 a c+b^2}}}$$
$$+\frac{\arctan\left(\frac{x \sqrt{2}\, \sqrt{c}}{\sqrt{b+\sqrt{-4 a c+b^2}}}\right) \sqrt{c}\, \left(b c d-b^2 e+a c e+\frac{-3 a b c e+2 a c^2 d+b^3 e-b^2 c d}{\sqrt{-4 a c+b^2}}\right) \sqrt{2}}{2 a^2 \left(a e^2-b d e+c d^2\right) \sqrt{b+\sqrt{-4 a c+b^2}}}$$

Result(type 3, 1159 leaves):

$$
-\frac{c^2\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)e}{2\left(a e^2-b d e+c d^2\right)a\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{c\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)b^2 e}{2\left(a e^2-b d e+c d^2\right)a^2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}
$$

$$
-\frac{c^2\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)b d}{2\left(a e^2-b d e+c d^2\right)a^2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{3c^2\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)b e}{2\left(a e^2-b d e+c d^2\right)a\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}
$$

$$
+\frac{c^3\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)d}{\left(a e^2-b d e+c d^2\right)a\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{c\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)b^3 e}{2\left(a e^2-b d e+c d^2\right)a^2\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}
$$

$$
-\frac{c^2\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)b^2 d}{2\left(a e^2-b d e+c d^2\right)a^2\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{c^2\sqrt{2}\,\arctan\left(\frac{c x\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)e}{2\left(a e^2-b d e+c d^2\right)a\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}
$$

$$
-\frac{c\sqrt{2}\,\arctan\left(\frac{c x\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)b^2 e}{2\left(a e^2-b d e+c d^2\right)a^2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{c^2\sqrt{2}\,\arctan\left(\frac{c x\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)b d}{2\left(a e^2-b d e+c d^2\right)a^2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}
$$

$$
-\frac{3c^2\sqrt{2}\,\arctan\left(\frac{c x\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)b e}{2\left(a e^2-b d e+c d^2\right)a\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{c^3\sqrt{2}\,\arctan\left(\frac{c x\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)d}{\left(a e^2-b d e+c d^2\right)a\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}
$$

$$
+\frac{c\sqrt{2}\,\arctan\left(\frac{c x\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)b^3 e}{2\left(a e^2-b d e+c d^2\right)a^2\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{c^2\sqrt{2}\,\arctan\left(\frac{c x\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)b^2 d}{2\left(a e^2-b d e+c d^2\right)a^2\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{1}{3 a d x^3}
$$

$$
+\frac{e}{a d^2 x}+\frac{b}{d a^2 x}+\frac{e^4\arctan\left(\frac{e x}{\sqrt{d e}}\right)}{d^2\left(a e^2-b d e+c d^2\right)\sqrt{d e}}
$$

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Problem 83: Result more than twice size of optimal antiderivative.
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$$\int \frac{x^3 \sqrt{c x^4 + b x^2 + a}}{e x^2 + d}\, dx$$

Optimal(type 3, 184 leaves, 7 steps):

$$\frac{\left(8 c^2 d^2 - b^2 e^2 - 4 c e \left(-a e + b d\right)\right) \operatorname{arctanh}\left(\frac{2 c x^2 + b}{2 \sqrt{c}\, \sqrt{c x^4 + b x^2 + a}}\right)}{16 c^{3/2} e^3} - \frac{d \operatorname{arctanh}\left(\frac{b d - 2 a e + \left(-b e + 2 c d\right) x^2}{2 \sqrt{a e^2 - b d e + c d^2}\, \sqrt{c x^4 + b x^2 + a}}\right) \sqrt{a e^2 - b d e + c d^2}}{2 e^3}$$

$$- \frac{\left(-2 c e x^2 - b e + 4 c d\right) \sqrt{c x^4 + b x^2 + a}}{8 c e^2}$$

Result(type 3, 886 leaves):

$$\frac{\sqrt{c x^4 + b x^2 + a}\, x^2}{4 e} + \frac{\sqrt{c x^4 + b x^2 + a}\, b}{8 e c} + \frac{\ln\left(\frac{\frac{b}{2} + c x^2}{\sqrt{c}} + \sqrt{c x^4 + b x^2 + a}\right) a}{4 e \sqrt{c}} - \frac{\ln\left(\frac{\frac{b}{2} + c x^2}{\sqrt{c}} + \sqrt{c x^4 + b x^2 + a}\right) b^2}{16 e c^{3/2}}$$

$$- \frac{d \sqrt{\left(x^2 + \frac{d}{e}\right)^2 c + \frac{\left(b e - 2 c d\right)\left(x^2 + \frac{d}{e}\right)}{e} + \frac{a e^2 - b d e + c d^2}{e^2}}}{2 e^2}$$

$$- \frac{d \ln\left(\frac{\frac{b e - 2 c d}{2 e} + c\left(x^2 + \frac{d}{e}\right)}{\sqrt{c}} + \sqrt{\left(x^2 + \frac{d}{e}\right)^2 c + \frac{\left(b e - 2 c d\right)\left(x^2 + \frac{d}{e}\right)}{e} + \frac{a e^2 - b d e + c d^2}{e^2}}\right) b}{4 e^2 \sqrt{c}}$$

$$+ \frac{d^2 \ln\left(\frac{\frac{b e - 2 c d}{2 e} + c\left(x^2 + \frac{d}{e}\right)}{\sqrt{c}} + \sqrt{\left(x^2 + \frac{d}{e}\right)^2 c + \frac{\left(b e - 2 c d\right)\left(x^2 + \frac{d}{e}\right)}{e} + \frac{a e^2 - b d e + c d^2}{e^2}}\right) \sqrt{c}}{2 e^3}$$

$$+ \frac{1}{2 e^2 \sqrt{\frac{a e^2 - b d e + c d^2}{e^2}}} \left(d \ln\left(\frac{1}{x^2 + \frac{d}{e}} \left(\frac{2\left(a e^2 - b d e + c d^2\right)}{e^2} + \frac{\left(b e - 2 c d\right)\left(x^2 + \frac{d}{e}\right)}{e} \right.\right.\right.$$

$$\left.\left.\left. + 2 \sqrt{\frac{a e^2 - b d e + c d^2}{e^2}} \sqrt{\left(x^2 + \frac{d}{e}\right)^2 c + \frac{\left(b e - 2 c d\right)\left(x^2 + \frac{d}{e}\right)}{e} + \frac{a e^2 - b d e + c d^2}{e^2}} \right)\right) a \right)$$

$$-\frac{1}{2e^3\sqrt{\frac{ae^2-bde+cd^2}{e^2}}}\left(d^2\ln\left(\frac{1}{x^2+\frac{d}{e}}\left(\frac{2(ae^2-bde+cd^2)}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}\right.\right.\right.$$

$$\left.\left.\left.+2\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}\right)\right)b\right)$$

$$+\frac{1}{2e^4\sqrt{\frac{ae^2-bde+cd^2}{e^2}}}\left(d^3\ln\left(\frac{1}{x^2+\frac{d}{e}}\left(\frac{2(ae^2-bde+cd^2)}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}\right.\right.\right.$$

$$\left.\left.\left.+2\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}\right)\right)c\right)$$

Problem 92: Result more than twice size of optimal antiderivative.

$$\int\frac{x^5}{(ex^2+d)(cx^4+bx^2+a)^{3/2}}\,dx$$

Optimal(type 3, 156 leaves, 5 steps):

$$\frac{d^2\operatorname{arctanh}\left(\frac{bd-2ae+(-be+2cd)x^2}{2\sqrt{ae^2-bde+cd^2}\sqrt{cx^4+bx^2+a}}\right)}{2(ae^2-bde+cd^2)^{3/2}}+\frac{-a(-2ae+bd)-(-abe-2adc+b^2d)x^2}{(-4ac+b^2)(ae^2-bde+cd^2)\sqrt{cx^4+bx^2+a}}$$

Result(type 3, 612 leaves):

$$-\frac{bx^2}{e\sqrt{cx^4+bx^2+a}\,(4ac-b^2)}-\frac{2a}{e\sqrt{cx^4+bx^2+a}\,(4ac-b^2)}-\frac{2dx^2c}{e^2\sqrt{cx^4+bx^2+a}\,(4ac-b^2)}-\frac{db}{e^2\sqrt{cx^4+bx^2+a}\,(4ac-b^2)}$$

$$-\frac{2d^2c\sqrt{\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2c+\sqrt{-4ac+b^2}\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}}{e^2(-4ac+b^2)\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}$$

$$+\frac{2 d^2 c \sqrt{\left(x^2+\frac{b+\sqrt{-4 a c+b^2}}{2 c}\right)^2 c-\sqrt{-4 a c+b^2}\left(x^2+\frac{b+\sqrt{-4 a c+b^2}}{2 c}\right)}}{e^2\left(-4 a c+b^2\right)\left(e \sqrt{-4 a c+b^2}+b e-2 c d\right)\left(x^2+\frac{b}{2 c}+\frac{\sqrt{-4 a c+b^2}}{2 c}\right)}$$

$$+\left(2 d^2 c \ln \left(\frac{\frac{2\left(a e^2-b d e+c d^2\right)}{e^2}+\frac{(b e-2 c d)\left(x^2+\frac{d}{e}\right)}{e}+2 \sqrt{\frac{a e^2-b d e+c d^2}{e^2}} \sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(b e-2 c d)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-b d e+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right.\right.$$

$$\left.\left.-2 c d\right) \sqrt{\frac{a e^2-b d e+c d^2}{e^2}}\right)$$

Problem 93: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\left(e x^2+d\right)\left(c x^4+b x^2+a\right)^{3/2}}\, dx$$

Optimal(type 3, 154 leaves, 5 steps):

$$\frac{e^2 \operatorname{arctanh}\left(\frac{b d-2 a e+(-b e+2 c d) x^2}{2 \sqrt{a e^2-b d e+c d^2} \sqrt{c x^4+b x^2+a}}\right)}{2\left(a e^2-b d e+c d^2\right)^{3/2}}+\frac{-b c d+b^2 e-2 a c e-c(-b e+2 c d) x^2}{\left(-4 a c+b^2\right)\left(a e^2-b d e+c d^2\right) \sqrt{c x^4+b x^2+a}}$$

Result(type 3, 453 leaves):

$$-\frac{2 c \sqrt{\left(x^2-\frac{-b+\sqrt{-4 a c+b^2}}{2 c}\right)^2 c+\sqrt{-4 a c+b^2}\left(x^2-\frac{-b+\sqrt{-4 a c+b^2}}{2 c}\right)}}{\left(-4 a c+b^2\right)\left(e \sqrt{-4 a c+b^2}-b e+2 c d\right)\left(x^2-\frac{-b+\sqrt{-4 a c+b^2}}{2 c}\right)}$$

$$+\frac{2 c \sqrt{\left(x^2+\frac{b+\sqrt{-4 a c+b^2}}{2 c}\right)^2 c-\sqrt{-4 a c+b^2}\left(x^2+\frac{b+\sqrt{-4 a c+b^2}}{2 c}\right)}}{\left(-4 a c+b^2\right)\left(e \sqrt{-4 a c+b^2}+b e-2 c d\right)\left(x^2+\frac{b+\sqrt{-4 a c+b^2}}{2 c}\right)}$$

$$+\left(2 c e \ln \left(\frac{\frac{2\left(a e^2-b d e+c d^2\right)}{e^2}+\frac{(b e-2 c d)\left(x^2+\frac{d}{e}\right)}{e}+2 \sqrt{\frac{a e^2-b d e+c d^2}{e^2}} \sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(b e-2 c d)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-b d e+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right.\right.$$

$$\sqrt{\frac{a e^2 - b d e + c d^2}{e^2}}\Bigg)$$

Problem 94: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 (e x^2 + d) (c x^4 + b x^2 + a)^{3/2}} \, dx$$

Optimal(type 3, 379 leaves, 15 steps):

$$\frac{3 b \operatorname{arctanh}\left(\frac{b x^2 + 2 a}{2 \sqrt{a} \sqrt{c x^4 + b x^2 + a}}\right)}{4 a^{5/2} d} + \frac{e \operatorname{arctanh}\left(\frac{b x^2 + 2 a}{2 \sqrt{a} \sqrt{c x^4 + b x^2 + a}}\right)}{2 a^{3/2} d^2} + \frac{e^4 \operatorname{arctanh}\left(\frac{b d - 2 a e + (-b e + 2 c d) x^2}{2 \sqrt{a e^2 - b d e + c d^2} \sqrt{c x^4 + b x^2 + a}}\right)}{2 d^2 (a e^2 - b d e + c d^2)^{3/2}}$$
$$- \frac{e (c x^2 b - 2 a c + b^2)}{a (-4 a c + b^2) d^2 \sqrt{c x^4 + b x^2 + a}} + \frac{c x^2 b - 2 a c + b^2}{a (-4 a c + b^2) d x^2 \sqrt{c x^4 + b x^2 + a}} - \frac{e^2 (b c d - b^2 e + 2 a c e + c (-b e + 2 c d) x^2)}{(-4 a c + b^2) d^2 (a e^2 - b d e + c d^2) \sqrt{c x^4 + b x^2 + a}}$$
$$- \frac{(-8 a c + 3 b^2) \sqrt{c x^4 + b x^2 + a}}{2 a^2 (-4 a c + b^2) d x^2}$$

Result(type 3, 862 leaves):

$$- \frac{1}{2 d a x^2 \sqrt{c x^4 + b x^2 + a}} - \frac{3 b}{4 d a^2 \sqrt{c x^4 + b x^2 + a}} + \frac{3 b^2 c x^2}{2 d a^2 \sqrt{c x^4 + b x^2 + a} (4 a c - b^2)} + \frac{3 b^3}{4 d a^2 \sqrt{c x^4 + b x^2 + a} (4 a c - b^2)}$$
$$+ \frac{3 b \ln\left(\frac{2 a + b x^2 + 2 \sqrt{a} \sqrt{c x^4 + b x^2 + a}}{x^2}\right)}{4 d a^{5/2}} - \frac{4 c^2 x^2}{d a \sqrt{c x^4 + b x^2 + a} (4 a c - b^2)} - \frac{2 c b}{d a \sqrt{c x^4 + b x^2 + a} (4 a c - b^2)}$$
$$- \frac{2 e^2 c \sqrt{\left(x^2 - \frac{-b + \sqrt{-4 a c + b^2}}{2 c}\right)^2 c + \sqrt{-4 a c + b^2} \left(x^2 - \frac{-b + \sqrt{-4 a c + b^2}}{2 c}\right)}}{d^2 (-4 a c + b^2) \left(e \sqrt{-4 a c + b^2} - b e + 2 c d\right) \left(x^2 + \frac{b}{2 c} - \frac{\sqrt{-4 a c + b^2}}{2 c}\right)}$$
$$+ \frac{2 e^2 c \sqrt{\left(x^2 + \frac{b + \sqrt{-4 a c + b^2}}{2 c}\right)^2 c - \sqrt{-4 a c + b^2} \left(x^2 + \frac{b + \sqrt{-4 a c + b^2}}{2 c}\right)}}{d^2 (-4 a c + b^2) \left(e \sqrt{-4 a c + b^2} + b e - 2 c d\right) \left(x^2 + \frac{b}{2 c} + \frac{\sqrt{-4 a c + b^2}}{2 c}\right)}$$

$$+\left(2 e^3 c \ln\left(\frac{\frac{2\left(a e^2-b d e+c d^2\right)}{e^2}+\frac{(b e-2 c d)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a e^2-b d e+c d^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2 c+\frac{(b e-2 c d)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-b d e+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right.\right.$$

$$\left.\left.-2 c d\right)\sqrt{\frac{a e^2-b d e+c d^2}{e^2}}\right)-\frac{e}{2 d^2 a\sqrt{c x^4+b x^2+a}}+\frac{e b x^2 c}{d^2 a\sqrt{c x^4+b x^2+a}\,(4 a c-b^2)}+\frac{e b^2}{2 d^2 a\sqrt{c x^4+b x^2+a}\,(4 a c-b^2)}$$

$$+\frac{e\ln\left(\frac{2 a+b x^2+2\sqrt{a}\sqrt{c x^4+b x^2+a}}{x^2}\right)}{2 d^2 a^{3/2}}$$

Problem 97: Result is not expressed in closed-form.

$$\int\frac{x^7\sqrt{e x^2+d}}{c x^4+b x^2+a}\,dx$$

Optimal(type 3, 355 leaves, 7 steps):

$$-\frac{(b e+c d)\left(e x^2+d\right)^{3/2}}{3 c^2 e^2}+\frac{\left(e x^2+d\right)^{5/2}}{5 c e^2}+\frac{\left(-a c+b^2\right)\sqrt{e x^2+d}}{c^3}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e x^2+d}}{\sqrt{2 c d-e\left(b-\sqrt{-4 a c+b^2}\right)}}\right)\left(b^2 c d-a c^2 d-b^3 e+2 a b c e+\frac{2 a^2 e c^2-4 a b^2 c e+3 a b d c^2+b^4 e-b^3 c d}{\sqrt{-4 a c+b^2}}\right)\sqrt{2}}{2 c^{7/2}\sqrt{2 c d-e\left(b-\sqrt{-4 a c+b^2}\right)}}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e x^2+d}}{\sqrt{2 c d-e\left(b+\sqrt{-4 a c+b^2}\right)}}\right)\left(b^2 c d-a c^2 d-b^3 e+2 a b c e+\frac{-2 a^2 e c^2+4 a b^2 c e-3 a b d c^2-b^4 e+b^3 c d}{\sqrt{-4 a c+b^2}}\right)\sqrt{2}}{2 c^{7/2}\sqrt{2 c d-e\left(b+\sqrt{-4 a c+b^2}\right)}}$$

Result(type 7, 495 leaves):

$$\frac{x^2\left(e x^2+d\right)^{3/2}}{5 c e}-\frac{2 d\left(e x^2+d\right)^{3/2}}{15 c e^2}-\frac{b\left(e x^2+d\right)^{3/2}}{3 c^2 e}+\frac{\sqrt{e}\,x a}{2 c^2}-\frac{\sqrt{e}\,x b^2}{2 c^3}-\frac{\sqrt{e x^2+d}\,a}{2 c^2}+\frac{\sqrt{e x^2+d}\,b^2}{2 c^3}-\frac{d a}{2 c^2\left(\sqrt{e x^2+d}-\sqrt{e}\,x\right)}$$

$$+\frac{d b^2}{2 c^3\left(\sqrt{e x^2+d}-\sqrt{e}\,x\right)}-\frac{1}{4 c^3}\left(\sum_{\_R=RootOf\left(c\_Z^8+(4 b e-4 c d)\_Z^6+(16 a e^2-8 b d e+6 c d^2)\_Z^4+(4 b d^2 e-4 c d^3)\_Z^2+c d^4\right)}\left(\left(\left(-2 a b c e+a c^2 d\right.\right.\right.\right.$$

$$+b^3 e-b^2 c d)\_R^6+\left(-4 a^2 c e^2+4 a b^2 e^2+2 a b c d e-3 a c^2 d^2-3 b^3 d e+3 b^2 c d^2\right)\_R^4+d\left(4 a^2 c e^2-4 a b^2 e^2-2 a b c d e+3 a c^2 d^2+3 b^3 d e\right.$$

$$\left.-3 b^2 c d^2\right)\_R^2+2 a b c d^3 e-a c^2 d^4-b^3 d^3 e+b^2 c d^4)\ln\left(\sqrt{e x^2+d}-\sqrt{e}\,x-\_R\right)\Big)\Big/\left(\_R^7 c+3\_R^5 b e-3\_R^5 c d+8\_R^3 a e^2-4\_R^3 b d e\right.$$

$$+3\_R^3 c d^2+\_R b d^2 e-\_R c d^3))$$

Problem 98: Result is not expressed in closed-form.

$$\int \frac{\sqrt{e x^2+d}}{c x^4+b x^2+a}\,dx$$

Optimal(type 3, 200 leaves, 11 steps):

$$\frac{\arctan\left(\frac{x\sqrt{2 c d-e\left(b-\sqrt{-4 a c+b^2}\right)}}{\sqrt{e x^2+d}\,\sqrt{b-\sqrt{-4 a c+b^2}}}\right)\sqrt{2 c d-e\left(b-\sqrt{-4 a c+b^2}\right)}}{\sqrt{-4 a c+b^2}\,\sqrt{b-\sqrt{-4 a c+b^2}}}-\frac{\arctan\left(\frac{x\sqrt{2 c d-e\left(b+\sqrt{-4 a c+b^2}\right)}}{\sqrt{e x^2+d}\,\sqrt{b+\sqrt{-4 a c+b^2}}}\right)\sqrt{2 c d-e\left(b+\sqrt{-4 a c+b^2}\right)}}{\sqrt{-4 a c+b^2}\,\sqrt{b+\sqrt{-4 a c+b^2}}}$$

Result(type 7, 160 leaves):

$$-\frac{1}{2}\left(e^{3/2}\left(\sum_{\_R=RootOf\left(c\_Z^4+(4 b e-4 c d)\_Z^3+(16 a e^2-8 b d e+6 c d^2)\_Z^2+(4 b d^2 e-4 c d^3)\_Z+c d^4\right)}\frac{(\_R^2+2\_R d+d^2)\ln\left(\left(\sqrt{e x^2+d}-\sqrt{e}\,x\right)^2-\_R\right)}{\_R^3 c+3\_R^2 b e-3\_R^2 c d+8\_R a e^2-4\_R b d e+3\_R c d^2+b d^2 e-c d^3}\right)\right)$$

Problem 99: Result is not expressed in closed-form.

$$\int \frac{\sqrt{e x^2+d}}{x^2\left(c x^4+b x^2+a\right)}\,dx$$

Optimal(type 3, 249 leaves, 8 steps):

$$-\frac{\sqrt{e x^2+d}}{a x}-\frac{c\arctan\left(\frac{x\sqrt{2 c d-e\left(b-\sqrt{-4 a c+b^2}\right)}}{\sqrt{e x^2+d}\,\sqrt{b-\sqrt{-4 a c+b^2}}}\right)\left(d+\frac{-2 a e+b d}{\sqrt{-4 a c+b^2}}\right)}{a\sqrt{2 c d-e\left(b-\sqrt{-4 a c+b^2}\right)}\,\sqrt{b-\sqrt{-4 a c+b^2}}}-\frac{c\arctan\left(\frac{x\sqrt{2 c d-e\left(b+\sqrt{-4 a c+b^2}\right)}}{\sqrt{e x^2+d}\,\sqrt{b+\sqrt{-4 a c+b^2}}}\right)\left(d+\frac{2 a e-b d}{\sqrt{-4 a c+b^2}}\right)}{a\sqrt{b+\sqrt{-4 a c+b^2}}\,\sqrt{2 c d-e\left(b+\sqrt{-4 a c+b^2}\right)}}$$

Result(type 7, 271 leaves):

$$-\frac{(e x^2+d)^{3/2}}{a d x}+\frac{e x\sqrt{e x^2+d}}{a d}+\frac{\sqrt{e}\ln\left(\sqrt{e}\,x+\sqrt{e x^2+d}\right)}{a}+\frac{1}{2a}\left(\sqrt{e}\left(\sum_{\_R=RootOf\left(c\_Z^4+(4be-4cd)\_Z^3+(16ae^2-8bde+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z+cd^4\right)}\frac{\left(\_R^2cd+2\left(-2ae^2+2bde-cd^2\right)\_R+cd^3\right)\ln\left(\left(\sqrt{e x^2+d}-\sqrt{e}\,x\right)^2-\_R\right)}{\_R^3c+3\_R^2be-3\_R^2cd+8\_Rae^2-4\_Rbde+3\_Rcd^2+bd^2e-cd^3}\right)\right)+\frac{\sqrt{e}\ln\left(\sqrt{e x^2+d}-\sqrt{e}\,x\right)}{a}$$

Problem 100: Result is not expressed in closed-form.

$$\int\frac{(e x^2+d)^{3/2}}{x\,(c x^4+b x^2+a)}\,dx$$

Optimal(type 3, 292 leaves, 8 steps):

$$-\frac{d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e x^2+d}}{\sqrt{d}}\right)}{a}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e x^2+d}}{\sqrt{2cd-e\left(b-\sqrt{-4ac+b^2}\right)}}\right)\left(-b\left(ae^2+cd^2\right)+ae^2\sqrt{-4ac+b^2}-cd\left(-4ae+d\sqrt{-4ac+b^2}\right)\right)\sqrt{2}}{2a\sqrt{c}\sqrt{-4ac+b^2}\sqrt{2cd-e\left(b-\sqrt{-4ac+b^2}\right)}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e x^2+d}}{\sqrt{2cd-e\left(b+\sqrt{-4ac+b^2}\right)}}\right)\left(b\left(ae^2+cd^2\right)+ae^2\sqrt{-4ac+b^2}-cd\left(4ae+d\sqrt{-4ac+b^2}\right)\right)\sqrt{2}}{2a\sqrt{c}\sqrt{-4ac+b^2}\sqrt{2cd-e\left(b+\sqrt{-4ac+b^2}\right)}}$$

Result(type 7, 387 leaves):

$$\frac{7(e x^2+d)^{3/2}}{24a}-\frac{d^{3/2}\ln\left(\frac{2d+2\sqrt{d}\sqrt{e x^2+d}}{x}\right)}{a}+\frac{3\sqrt{e x^2+d}\,d}{8a}+\frac{e^{3/2}x^3}{6a}-\frac{e\sqrt{e x^2+d}\,x^2}{8a}+\frac{3\sqrt{e}\,x d}{4a}-\frac{1}{4a}\left(\sum_{\_R=RootOf\left(c\_Z^8+(4be-4cd)\_Z^6+(16ae^2-8bde+6cd^2)\_Z^4+(4bd^2e-4cd^3)\_Z^2+cd^4\right)}\frac{\left(\left(-ae^2+cd^2\right)\_R^6+d\left(-5ae^2+4bde-3cd^2\right)\_R^4+d^2\left(5ae^2-4bde+3cd^2\right)\_R^2+ad^3e^2-cd^5\right)\ln\left(\sqrt{e x^2+d}-\sqrt{e}\,x-\_R\right)}{\_R^7c+3\_R^5be-3\_R^5cd+8\_R^3ae^2-4\_R^3bde+3\_R^3cd^2+\_Rbd^2e-\_Rcd^3}\right)$$
$$-\frac{5d^2}{8a\left(\sqrt{e x^2+d}-\sqrt{e}\,x\right)}-\frac{d^3}{24a\left(\sqrt{e x^2+d}-\sqrt{e}\,x\right)^3}$$

Problem 101: Result is not expressed in closed-form.

$$\int \frac{(e x^2+d)^{3/2}}{x^4 (c x^4+b x^2+a)}\, dx$$

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Optimal(type 3, 445 leaves, 19 steps):
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$$-\frac{(e x^2+d)^{3/2}}{3 a x^3}-\frac{(-a e+b d)\,\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e x^2+d}}\right)\sqrt{e}}{a^2}+\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e x^2+d}}\right)\left(b d-a e+\frac{a b e+2 a d c-b^2 d}{\sqrt{-4 a c+b^2}}\right)\sqrt{e}}{2 a^2}$$

$$+\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e x^2+d}}\right)\left(b d-a e+\frac{-a b e-2 a d c+b^2 d}{\sqrt{-4 a c+b^2}}\right)\sqrt{e}}{2 a^2}+\frac{(-a e+b d)\sqrt{e x^2+d}}{x a^2}$$

$$+\frac{\arctan\left(\frac{x\sqrt{2 c d-e\left(b-\sqrt{-4 a c+b^2}\right)}}{\sqrt{e x^2+d}\sqrt{b-\sqrt{-4 a c+b^2}}}\right)\left(b d-a e+\frac{-a b e-2 a d c+b^2 d}{\sqrt{-4 a c+b^2}}\right)\sqrt{2 c d-e\left(b-\sqrt{-4 a c+b^2}\right)}}{2 a^2\sqrt{b-\sqrt{-4 a c+b^2}}}$$

$$+\frac{\arctan\left(\frac{x\sqrt{2 c d-e\left(b+\sqrt{-4 a c+b^2}\right)}}{\sqrt{e x^2+d}\sqrt{b+\sqrt{-4 a c+b^2}}}\right)\left(b d-a e+\frac{a b e+2 a d c-b^2 d}{\sqrt{-4 a c+b^2}}\right)\sqrt{2 c d-e\left(b+\sqrt{-4 a c+b^2}\right)}}{2 a^2\sqrt{b+\sqrt{-4 a c+b^2}}}$$

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Result(type 7, 510 leaves):
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$$-\frac{(e x^2+d)^{5/2}}{3 a d x^3}-\frac{2 e (e x^2+d)^{5/2}}{3 a d^2 x}+\frac{2 e^2 x (e x^2+d)^{3/2}}{3 a d^2}+\frac{e^2 x\sqrt{e x^2+d}}{a d}+\frac{e^{3/2}\ln\left(\sqrt{e}\,x+\sqrt{e x^2+d}\right)}{a}-\frac{e^{3/2} x^2 b}{4 a^2}-\frac{5 e\sqrt{e x^2+d}\,x b}{4 a^2}-\frac{\sqrt{e}\,b d}{8 a^2}$$

$$+\frac{1}{2 a^2}\left(\sqrt{e}\left(\sum_{\_R=RootOf(c\_Z^4+(4 b e-4 c d)\_Z^3+(16 a e^2-8 b d e+6 c d^2)\_Z^2+(4 b d^2 e-4 c d^3)\_Z+c d^4)}\frac{(c d (2 a e-b d)\_R^2+2(-2 a^2 e^3+4 a d e^2 b-2 d^2 e b^2+b c d^3)\_R+2 a c d^3 e-b c d^4)\ln\left(\left(\sqrt{e x^2+d}-\sqrt{e}\,x\right)^2-\_R\right)}{\_R^3 c+3\_R^2 b e-3\_R^2 c d+8\_R a e^2-4\_R b d e+3\_R c d^2+b d^2 e-c d^3}\right)\right)$$

$$+\frac{e^{3/2}\ln\left(\sqrt{e x^2+d}-\sqrt{e}\,x\right)}{a}-\frac{3\sqrt{e}\ln\left(\sqrt{e x^2+d}-\sqrt{e}\,x\right) b d}{2 a^2}+\frac{\sqrt{e}\,b d^2}{8 a^2\left(\sqrt{e x^2+d}-\sqrt{e}\,x\right)^2}+\frac{b (e x^2+d)^{5/2}}{a^2 d x}-\frac{b e x (e x^2+d)^{3/2}}{a^2 d}$$

$$-\frac{3 b\sqrt{e}\,d\ln\left(\sqrt{e}\,x+\sqrt{e x^2+d}\right)}{2 a^2}$$

Problem 102: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{-x^2+1}}{x^3\left(c x^4+b x^2+a\right)}\,dx$$

Optimal(type 3, 240 leaves, 8 steps):

$$\frac{(a+2b)\operatorname{arctanh}\left(\sqrt{-x^2+1}\right)}{2a^2}-\frac{1}{4a\left(1-\sqrt{-x^2+1}\right)}+\frac{1}{4a\left(1+\sqrt{-x^2+1}\right)}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{-x^2+1}}{\sqrt{b+2c-\sqrt{-4ac+b^2}}}\right)\sqrt{c}\left(a+b+\frac{b^2+a(b-2c)}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{2a^2\sqrt{b+2c-\sqrt{-4ac+b^2}}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{-x^2+1}}{\sqrt{b+2c+\sqrt{-4ac+b^2}}}\right)\sqrt{c}\left(a+b+\frac{-b^2-a(b-2c)}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{2a^2\sqrt{b+2c+\sqrt{-4ac+b^2}}}$$

Result(type ?, 2769 leaves): Display of huge result suppressed!

Problem 103: Result is not expressed in closed-form.

$$\int \frac{x^2\sqrt{-x^2+1}}{c x^4+b x^2+a}\,dx$$

Optimal(type 3, 225 leaves, 8 steps):

$$-\frac{\arcsin(x)}{c}+\frac{\arctan\left(\frac{x\sqrt{b+2c-\sqrt{-4ac+b^2}}}{\sqrt{-x^2+1}\sqrt{b-\sqrt{-4ac+b^2}}}\right)\left(b+c+\frac{2ac-b^2-bc}{\sqrt{-4ac+b^2}}\right)}{c\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{b+2c-\sqrt{-4ac+b^2}}}$$
$$+\frac{\arctan\left(\frac{x\sqrt{b+2c+\sqrt{-4ac+b^2}}}{\sqrt{-x^2+1}\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(b+c+\frac{-2ac+b^2+bc}{\sqrt{-4ac+b^2}}\right)}{c\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{b+2c+\sqrt{-4ac+b^2}}}$$

Result(type 7, 174 leaves):

$$\frac{2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)}{c}$$

$$-\frac{1}{4c}\left(\sum_{\_R=RootOf(a\_Z^8+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a)} \frac{(\_R^6 a+(4c+3a+4b)\_R^4+(4c+3a+4b)\_R^2+a)\ln\left(\frac{\sqrt{-x^2+1}-1}{x}-\_R\right)}{\_R^7 a+3\_R^5 a+3\_R^5 b+3\_R^3 a+4\_R^3 b+8\_R^3 c+\_R a+\_R b}\right)$$

Problem 104: Result is not expressed in closed-form.

$$\int \frac{\sqrt{-x^2+1}}{c x^4+b x^2+a}\, dx$$

Optimal(type 3, 180 leaves, 9 steps):

$$\frac{\arctan\left(\frac{x\sqrt{b+2c-\sqrt{-4ac+b^2}}}{\sqrt{-x^2+1}\sqrt{b-\sqrt{-4ac+b^2}}}\right)\sqrt{b+2c-\sqrt{-4ac+b^2}}}{\sqrt{-4ac+b^2}\sqrt{b-\sqrt{-4ac+b^2}}}-\frac{\arctan\left(\frac{x\sqrt{b+2c+\sqrt{-4ac+b^2}}}{\sqrt{-x^2+1}\sqrt{b+\sqrt{-4ac+b^2}}}\right)\sqrt{b+2c+\sqrt{-4ac+b^2}}}{\sqrt{-4ac+b^2}\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type 7, 129 leaves):

$$-\frac{\left(\sum_{\_R=RootOf(a\_Z^8+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a)} \frac{(\_R^6-\_R^4-\_R^2+1)\ln\left(\frac{\sqrt{-x^2+1}-1}{x}-\_R\right)}{\_R^7 a+3\_R^5 a+3\_R^5 b+3\_R^3 a+4\_R^3 b+8\_R^3 c+\_R a+\_R b}\right)}{4}$$

Problem 105: Result is not expressed in closed-form.

$$\int \frac{x^4}{(c x^4+b x^2+a)\sqrt{e x^2+d}}\, dx$$

Optimal(type 3, 253 leaves, 10 steps):

$$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e x^2+d}}\right)}{c\sqrt{e}}-\frac{\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{-4ac+b^2}\right)}}{\sqrt{e x^2+d}\sqrt{b-\sqrt{-4ac+b^2}}}\right)\left(b+\frac{2ac-b^2}{\sqrt{-4ac+b^2}}\right)}{c\sqrt{2cd-e\left(b-\sqrt{-4ac+b^2}\right)}\sqrt{b-\sqrt{-4ac+b^2}}}$$

$$-\frac{\arctan\left(\frac{x\sqrt{2cd-e\left(b+\sqrt{-4ac+b^2}\right)}}{\sqrt{ex^2+d}\,\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(b+\frac{-2ac+b^2}{\sqrt{-4ac+b^2}}\right)}{c\sqrt{b+\sqrt{-4ac+b^2}}\,\sqrt{2cd-e\left(b+\sqrt{-4ac+b^2}\right)}}$$

Result(type 7, 199 leaves):

$$\frac{\ln\left(\sqrt{e}\,x+\sqrt{ex^2+d}\right)}{c\sqrt{e}}+\frac{1}{2c}\left(\sqrt{e}\left(\sum_{\_R=RootOf\left(c\_Z^4+(4be-4cd)\_Z^3+(16ae^2-8bde+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z+cd^4\right)}\frac{\left(b\_R^2+2(2ae-bd)\_R+bd^2\right)\ln\left(\left(\sqrt{ex^2+d}-\sqrt{e}\,x\right)^2-\_R\right)}{\_R^3c+3\_R^2be-3\_R^2cd+8\_Rae^2-4\_Rbde+3\_Rcd^2+bd^2e-cd^3}\right)\right)$$

Problem 106: Result is not expressed in closed-form.

$$\int\frac{1}{(cx^4+bx^2+a)\sqrt{ex^2+d}}\,dx$$

Optimal(type 3, 203 leaves, 5 steps):

$$\frac{2c\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{-4ac+b^2}\right)}}{\sqrt{ex^2+d}\,\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{2cd-e\left(b-\sqrt{-4ac+b^2}\right)}\,\sqrt{b-\sqrt{-4ac+b^2}}}-\frac{2c\arctan\left(\frac{x\sqrt{2cd-e\left(b+\sqrt{-4ac+b^2}\right)}}{\sqrt{ex^2+d}\,\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{b+\sqrt{-4ac+b^2}}\,\sqrt{2cd-e\left(b+\sqrt{-4ac+b^2}\right)}}$$

Result(type 7, 150 leaves):

$$-2e^{3/2}\left(\sum_{\_R=RootOf\left(c\_Z^4+(4be-4cd)\_Z^3+(16ae^2-8bde+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z+cd^4\right)}\frac{\_R\ln\left(\left(\sqrt{ex^2+d}-\sqrt{e}\,x\right)^2-\_R\right)}{\_R^3c+3\_R^2be-3\_R^2cd+8\_Rae^2-4\_Rbde+3\_Rcd^2+bd^2e-cd^3}\right)$$

Problem 107: Result is not expressed in closed-form.

$$\int \frac{1}{x^4\left(c x^4+b x^2+a\right)\sqrt{e x^2+d}}\, dx$$

Optimal(type 3, 292 leaves, 11 steps):

$$-\frac{\sqrt{e x^2+d}}{3 a d x^3}+\frac{b\sqrt{e x^2+d}}{a^2 d x}+\frac{2 e\sqrt{e x^2+d}}{3 a d^2 x}+\frac{c \arctan\left(\frac{x\sqrt{2 c d-e\left(b-\sqrt{-4 a c+b^2}\right)}}{\sqrt{e x^2+d}\sqrt{b-\sqrt{-4 a c+b^2}}}\right)\left(b+\frac{-2 a c+b^2}{\sqrt{-4 a c+b^2}}\right)}{a^2\sqrt{2 c d-e\left(b-\sqrt{-4 a c+b^2}\right)}\sqrt{b-\sqrt{-4 a c+b^2}}}$$
$$+\frac{c \arctan\left(\frac{x\sqrt{2 c d-e\left(b+\sqrt{-4 a c+b^2}\right)}}{\sqrt{e x^2+d}\sqrt{b+\sqrt{-4 a c+b^2}}}\right)\left(b+\frac{2 a c-b^2}{\sqrt{-4 a c+b^2}}\right)}{a^2\sqrt{b+\sqrt{-4 a c+b^2}}\sqrt{2 c d-e\left(b+\sqrt{-4 a c+b^2}\right)}}$$

Result(type 7, 247 leaves):

$$-\frac{\sqrt{e x^2+d}}{3 a d x^3}+\frac{2 e\sqrt{e x^2+d}}{3 a d^2 x}-\frac{1}{2 a^2}\left(\sqrt{e}\left(\sum_{\_R=RootOf\left(c\_Z^4+(4 b e-4 c d)\_Z^3+(16 a e^2-8 b d e+6 c d^2)\_Z^2+(4 b d^2 e-4 c d^3)\_Z+c d^4\right)}\frac{\left(c\_R^2 b+2\left(-2 a c e+2 b^2 e-b c d\right)\_R+b c d^2\right)\ln\left(\left(\sqrt{e x^2+d}-\sqrt{e}\, x\right)^2-\_R\right)}{\_R^3 c+3\_R^2 b e-3\_R^2 c d+8\_R a e^2-4\_R b d e+3\_R c d^2+b d^2 e-c d^3}\right)\right)+\frac{b\sqrt{e x^2+d}}{a^2 d x}$$

Problem 108: Unable to integrate problem.

$$\int \frac{x^3\left(e x^2+d\right)^q}{c x^4+b x^2+a}\, dx$$

Optimal(type 5, 198 leaves, 5 steps):

$$-\frac{\left(e x^2+d\right)^{1+q}\mathrm{hypergeom}\left(\left[1, 1+q\right],\left[2+q\right],\frac{2 c\left(e x^2+d\right)}{2 c d-e\left(b-\sqrt{-4 a c+b^2}\right)}\right)\left(1-\frac{b}{\sqrt{-4 a c+b^2}}\right)}{2\left(1+q\right)\left(2 c d-e\left(b-\sqrt{-4 a c+b^2}\right)\right)}$$
$$-\frac{\left(e x^2+d\right)^{1+q}\mathrm{hypergeom}\left(\left[1, 1+q\right],\left[2+q\right],\frac{2 c\left(e x^2+d\right)}{2 c d-e\left(b+\sqrt{-4 a c+b^2}\right)}\right)\left(1+\frac{b}{\sqrt{-4 a c+b^2}}\right)}{2\left(1+q\right)\left(2 c d-e\left(b+\sqrt{-4 a c+b^2}\right)\right)}$$

Result(type 8, 29 leaves):

$$\int \frac{x^3 (e x^2+d)^q}{c x^4+b x^2+a}\, dx$$

Problem 109: Unable to integrate problem.

$$\int \frac{x^6 (e x^2+d)^q}{c x^4+b x^2+a}\, dx$$

Optimal(type 6, 313 leaves, 12 steps):

$$-\frac{b x (e x^2+d)^q \operatorname{hypergeom}\left(\left[\frac{1}{2}, -q\right], \left[\frac{3}{2}\right], -\frac{e x^2}{d}\right)}{c^2 \left(1+\frac{e x^2}{d}\right)^q} + \frac{x^3 (e x^2+d)^q \operatorname{hypergeom}\left(\left[\frac{3}{2}, -q\right], \left[\frac{5}{2}\right], -\frac{e x^2}{d}\right)}{3 c \left(1+\frac{e x^2}{d}\right)^q}$$
$$+ \frac{x (e x^2+d)^q \mathit{AppellF1}\left(\frac{1}{2}, -q, 1, \frac{3}{2}, -\frac{e x^2}{d}, -\frac{2 x^2 c}{b-\sqrt{-4 a c+b^2}}\right) \left(b^2-a c-\frac{b(-3 a c+b^2)}{\sqrt{-4 a c+b^2}}\right)}{c^2 \left(1+\frac{e x^2}{d}\right)^q \left(b-\sqrt{-4 a c+b^2}\right)}$$
$$+ \frac{x (e x^2+d)^q \mathit{AppellF1}\left(\frac{1}{2}, -q, 1, \frac{3}{2}, -\frac{e x^2}{d}, -\frac{2 x^2 c}{b+\sqrt{-4 a c+b^2}}\right) \left(b^2-a c+\frac{b(-3 a c+b^2)}{\sqrt{-4 a c+b^2}}\right)}{c^2 \left(1+\frac{e x^2}{d}\right)^q \left(b+\sqrt{-4 a c+b^2}\right)}$$

Result(type 8, 29 leaves):

$$\int \frac{x^6 (e x^2+d)^q}{c x^4+b x^2+a}\, dx$$

Test results for the 30 problems in "1.2.2.5 P(x) (a+b x^2+c x^4)^p.txt"

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \frac{h x^4+g x^3+f x^2+e x+d}{x^4+x^2+1}\, dx$$

Optimal(type 3, 115 leaves, 17 steps):

$$h x - \frac{(d-f) \ln(x^2-x+1)}{4} + \frac{(d-f) \ln(x^2+x+1)}{4} + \frac{g \ln(x^4+x^2+1)}{4} - \frac{(d+f-2 h) \arctan\left(\frac{(1-2 x)\sqrt{3}}{3}\right)\sqrt{3}}{6}$$
$$+ \frac{(d+f-2 h) \arctan\left(\frac{(2 x+1)\sqrt{3}}{3}\right)\sqrt{3}}{6} + \frac{(2 e-g) \arctan\left(\frac{(2 x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}$$

Result(type 3, 240 leaves):

$$hx+\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{4}+\frac{\ln\left(x^2+x+1\right)g}{4}+\frac{d\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)e}{3}$$
$$+\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)f}{6}+\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)g}{6}-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)h}{3}+\frac{\ln\left(x^2-x+1\right)f}{4}-\frac{d\ln\left(x^2-x+1\right)}{4}$$
$$+\frac{\ln\left(x^2-x+1\right)g}{4}+\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)d}{6}+\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)e}{3}+\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)f}{6}$$
$$-\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)g}{6}-\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)h}{3}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int\frac{ix^5+hx^4+gx^3+fx^2+ex+d}{x^4+x^2+1}\,dx$$

Optimal(type 3, 128 leaves, 19 steps):

$$hx+\frac{ix^2}{2}-\frac{(d-f)\ln\left(x^2-x+1\right)}{4}+\frac{(d-f)\ln\left(x^2+x+1\right)}{4}+\frac{(g-i)\ln\left(x^4+x^2+1\right)}{4}-\frac{(d+f-2h)\arctan\left(\frac{(1-2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}$$
$$+\frac{(d+f-2h)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{(2e-g-i)\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}$$

Result(type 3, 302 leaves):

$$\frac{ix^2}{2}+hx+\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{4}+\frac{\ln\left(x^2+x+1\right)g}{4}-\frac{\ln\left(x^2+x+1\right)i}{4}+\frac{d\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}$$
$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)e}{3}+\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)f}{6}+\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)g}{6}-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)h}{3}$$
$$+\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)i}{6}+\frac{\ln\left(x^2-x+1\right)g}{4}-\frac{\ln\left(x^2-x+1\right)i}{4}+\frac{\ln\left(x^2-x+1\right)f}{4}-\frac{d\ln\left(x^2-x+1\right)}{4}+\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)d}{6}$$
$$+\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)e}{3}+\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)f}{6}-\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)g}{6}-\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)h}{3}$$

$$-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right) i}{6}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \frac{h x^4 + g x^3 + f x^2 + e x + d}{c x^4 + b x^2 + a}\, dx$$

Optimal(type 3, 247 leaves, 11 steps):

$$\frac{h x}{c} + \frac{g \ln\left(c x^4 + b x^2 + a\right)}{4 c} - \frac{(-b g + 2 c e)\, \operatorname{arctanh}\left(\frac{2 c x^2 + b}{\sqrt{-4 a c + b^2}}\right)}{2 c \sqrt{-4 a c + b^2}} + \frac{\arctan\left(\frac{x \sqrt{2}\, \sqrt{c}}{\sqrt{b - \sqrt{-4 a c + b^2}}}\right) \left(c f - b h + \frac{2 c^2 d + b^2 h - c (2 a h + b f)}{\sqrt{-4 a c + b^2}}\right) \sqrt{2}}{2 c^{3/2} \sqrt{b - \sqrt{-4 a c + b^2}}}$$
$$+ \frac{\arctan\left(\frac{x \sqrt{2}\, \sqrt{c}}{\sqrt{b + \sqrt{-4 a c + b^2}}}\right) \left(c f - b h + \frac{2 a c h - b^2 h + b c f - 2 c^2 d}{\sqrt{-4 a c + b^2}}\right) \sqrt{2}}{2 c^{3/2} \sqrt{b + \sqrt{-4 a c + b^2}}}$$

Result(type 3, 1131 leaves):

$$\frac{h x}{c} - \frac{(-4 a c + b^2) \ln\left(-2 c x^2 + \sqrt{-4 a c + b^2} - b\right) g}{4 (4 a c - b^2) c} + \frac{\sqrt{-4 a c + b^2}\, \ln\left(-2 c x^2 + \sqrt{-4 a c + b^2} - b\right) b g}{4 (4 a c - b^2) c}$$
$$- \frac{\sqrt{-4 a c + b^2}\, \ln\left(-2 c x^2 + \sqrt{-4 a c + b^2} - b\right) e}{2 (4 a c - b^2)} - \frac{(-4 a c + b^2) \sqrt{2}\, \operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b + \sqrt{-4 a c + b^2}\right) c}}\right) b h}{2 (4 a c - b^2) c \sqrt{\left(-b + \sqrt{-4 a c + b^2}\right) c}}$$
$$+ \frac{(-4 a c + b^2) \sqrt{2}\, \operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b + \sqrt{-4 a c + b^2}\right) c}}\right) f}{2 (4 a c - b^2) \sqrt{\left(-b + \sqrt{-4 a c + b^2}\right) c}} - \frac{\sqrt{-4 a c + b^2}\, \sqrt{2}\, \operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b + \sqrt{-4 a c + b^2}\right) c}}\right) a h}{(4 a c - b^2) \sqrt{\left(-b + \sqrt{-4 a c + b^2}\right) c}}$$
$$+ \frac{\sqrt{-4 a c + b^2}\, \sqrt{2}\, \operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b + \sqrt{-4 a c + b^2}\right) c}}\right) b^2 h}{2 (4 a c - b^2) c \sqrt{\left(-b + \sqrt{-4 a c + b^2}\right) c}} - \frac{\sqrt{-4 a c + b^2}\, \sqrt{2}\, \operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b + \sqrt{-4 a c + b^2}\right) c}}\right) b f}{2 (4 a c - b^2) \sqrt{\left(-b + \sqrt{-4 a c + b^2}\right) c}}$$

$$+\frac{\sqrt{-4ac+b^2}\,c\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)d}{(4ac-b^2)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{(-4ac+b^2)\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)g}{4(4ac-b^2)c}$$

$$-\frac{\sqrt{-4ac+b^2}\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)bg}{4(4ac-b^2)c}+\frac{\sqrt{-4ac+b^2}\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)e}{2(4ac-b^2)}$$

$$+\frac{(-4ac+b^2)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)bh}{2(4ac-b^2)c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{(-4ac+b^2)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)f}{2(4ac-b^2)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

$$-\frac{\sqrt{-4ac+b^2}\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)ah}{(4ac-b^2)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{-4ac+b^2}\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)b^2h}{2(4ac-b^2)c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

$$-\frac{\sqrt{-4ac+b^2}\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)bf}{2(4ac-b^2)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{-4ac+b^2}\,c\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)d}{(4ac-b^2)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

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Problem 11: Result more than twice size of optimal antiderivative.
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$$\int\frac{hx^4+gx^3+fx^2+ex+d}{(x^4-5x^2+4)^2}\,dx$$

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Optimal(type 3, 136 leaves, 10 steps):
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$$\frac{5e+8g-(2e+5g)x^2}{18(x^4-5x^2+4)}+\frac{x\left(17d+20f+32h-(5d+8f+20h)x^2\right)}{72(x^4-5x^2+4)}+\frac{(19d+52f+112h)\operatorname{arctanh}\left(\frac{x}{2}\right)}{432}-\frac{(d+7f+13h)\operatorname{arctanh}(x)}{54}$$

$$+\frac{(2e+5g)\ln(-x^2+1)}{54}-\frac{(2e+5g)\ln(-x^2+4)}{54}$$

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Result(type 3, 301 leaves):
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$$-\frac{19\ln(x-2)d}{864}-\frac{\ln(x-2)e}{27}-\frac{\ln(2+x)e}{27}+\frac{19\ln(2+x)d}{864}+\frac{\ln(x-1)d}{108}+\frac{\ln(x-1)e}{27}-\frac{\ln(x+1)d}{108}+\frac{\ln(x+1)e}{27}-\frac{7\ln(x+1)f}{108}$$

$$-\frac{h}{36(x+1)}+\frac{g}{36(x+1)}-\frac{f}{36(x+1)}+\frac{e}{36(x+1)}-\frac{d}{36(x+1)}-\frac{13\ln(x-2)f}{216}-\frac{5\ln(x-2)g}{54}-\frac{7\ln(x-2)h}{54}-\frac{d}{144(x-2)}$$

$$-\frac{e}{72(x-2)}-\frac{f}{36(x-2)}-\frac{g}{18(x-2)}-\frac{h}{9(x-2)}-\frac{d}{144(2+x)}+\frac{e}{72(2+x)}-\frac{f}{36(2+x)}+\frac{g}{18(2+x)}-\frac{h}{9(2+x)}+\frac{13\ln(2+x)f}{216}$$

$$-\frac{5\ln(2+x)\,g}{54}+\frac{7\ln(2+x)\,h}{54}-\frac{d}{36\,(x-1)}-\frac{e}{36\,(x-1)}-\frac{f}{36\,(x-1)}-\frac{g}{36\,(x-1)}-\frac{h}{36\,(x-1)}+\frac{7\ln(x-1)\,f}{108}+\frac{5\ln(x-1)\,g}{54}+\frac{13\ln(x-1)\,h}{108}-\frac{13\ln(x+1)\,h}{108}+\frac{5\ln(x+1)\,g}{54}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{i x^5+h x^4+g x^3+f x^2+e x+d}{(x^4+x^2+1)^2}\,dx$$

Optimal(type 3, 171 leaves, 16 steps):

$$\frac{x\left(d+f-2h-(d-2f+h)\,x^2\right)}{6\,(x^4+x^2+1)}+\frac{e-2g+i+(2e-g-i)\,x^2}{6\,(x^4+x^2+1)}-\frac{(2d-f+h)\ln\left(x^2-x+1\right)}{8}+\frac{(2d-f+h)\ln\left(x^2+x+1\right)}{8}-\frac{(4d+f+h)\arctan\left(\frac{(1-2x)\sqrt{3}}{3}\right)\sqrt{3}}{36}+\frac{(4d+f+h)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{36}+\frac{(2e-g+2i)\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}$$

Result(type 3, 373 leaves):

$$\frac{\left(-\frac{d}{3}-\frac{h}{3}-\frac{e}{3}-\frac{g}{3}+\frac{2f}{3}+\frac{2i}{3}\right)x-\frac{2d}{3}+\frac{h}{3}+\frac{e}{3}-\frac{2g}{3}+\frac{f}{3}+\frac{i}{3}}{4\,(x^2+x+1)}+\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{8}+\frac{\ln\left(x^2+x+1\right)h}{8}+\frac{d\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{2\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)e}{9}+\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)f}{36}+\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)g}{9}+\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)h}{36}-\frac{2\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)i}{9}-\frac{\left(\frac{d}{3}+\frac{h}{3}-\frac{e}{3}-\frac{g}{3}-\frac{2f}{3}+\frac{2i}{3}\right)x-\frac{2d}{3}+\frac{h}{3}-\frac{e}{3}+\frac{2g}{3}+\frac{f}{3}-\frac{i}{3}}{4\,(x^2-x+1)}-\frac{d\ln\left(x^2-x+1\right)}{4}+\frac{\ln\left(x^2-x+1\right)f}{8}-\frac{\ln\left(x^2-x+1\right)h}{8}+\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)d}{9}+\frac{2\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)e}{9}+\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)f}{36}-\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)g}{9}+\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)h}{36}+\frac{2\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)i}{9}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \frac{h x^4+g x^3+f x^2+e x+d}{(c x^4+b x^2+a)^2}\,dx$$

Optimal(type 3, 393 leaves, 9 steps):

$$\frac{-be+2ag-(-bg+2ce)x^2}{2(-4ac+b^2)(cx^4+bx^2+a)}+\frac{x\left(b^2d-abf-2a(-ah+cd)+(abh-2acf+bcd)x^2\right)}{2a(-4ac+b^2)(cx^4+bx^2+a)}+\frac{(-bg+2ce)\operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{3/2}}$$

$$+\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\left(bcd-2acf+abh+\frac{4abcf+b^2(-ah+cd)-4ac(ah+3cd)}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4a(-4ac+b^2)\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}}$$

$$+\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(bcd-2acf+abh+\frac{-4abcf-b^2(-ah+cd)+4ac(ah+3cd)}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4a(-4ac+b^2)\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type ?, 7597 leaves): Display of huge result suppressed!

Problem 14: Result more than twice size of optimal antiderivative.

$$\int\frac{mx^8+lx^7+kx^6+jx^5+hx^4+gx^3+fx^2+ex+d}{(cx^4+bx^2+a)^2}\,dx$$

Optimal(type 3, 718 leaves, 13 steps):

$$\frac{mx}{c^2}+\frac{-bc(aj+ce)+ab^2l+2ac(-al+cg)-\left(2c^3e-c^2(2aj+bg)-b^3l+bc(3al+bj)\right)x^2}{2c^2(-4ac+b^2)(cx^4+bx^2+a)}$$

$$-\frac{x\left(abc(ak+cf)-b^2(a^2m+c^2d)+2ac(a^2m-ach+c^2d)+\left(ab^2ck+2ac^2(-ak+cf)-ab^3m-bc(-3a^2m+ach+c^2d)\right)x^2\right)}{2ac^2(-4ac+b^2)(cx^4+bx^2+a)}$$

$$+\frac{\left(4c^3e-c^2(-4aj+2bg)+b^3l-6abcl\right)\operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{2c^2(-4ac+b^2)^{3/2}}+\frac{l\ln(cx^4+bx^2+a)}{4c^2}$$

$$+\frac{1}{4ac^{5/2}(-4ac+b^2)\sqrt{b-\sqrt{-4ac+b^2}}}\left(\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\left(ab^2ck-2ac^2(3ak+cf)-3ab^3m+bc(13a^2m+ach+c^2d)\right.\right.$$
$$\left.\left.+\frac{-ab^3ck+4abc^2(2ak+cf)+3ab^4m+b^2c(-19a^2m-ach+c^2d)-4ac^2(-5a^2m+ach+3c^2d)}{\sqrt{-4ac+b^2}}\right)\sqrt{2}\right)$$

$$+\frac{1}{4ac^{5/2}(-4ac+b^2)\sqrt{b+\sqrt{-4ac+b^2}}}\left(\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(ab^2ck-2ac^2(3ak+cf)-3ab^3m+bc(13a^2m+ach+c^2d)\right.\right.$$
$$\left.\left.+\frac{ab^3ck-4abc^2(2ak+cf)-3ab^4m-b^2c(-19a^2m-ach+c^2d)+4ac^2(-5a^2m+ach+3c^2d)}{\sqrt{-4ac+b^2}}\right)\sqrt{2}\right)$$

Result(type ?, 16516 leaves): Display of huge result suppressed!

Problem 15: Result more than twice size of optimal antiderivative.

$$\int \frac{h x^4 + g x^3 + f x^2 + e x + d}{\left(x^4 - 5 x^2 + 4\right)^3}\, dx$$

Optimal(type 3, 206 leaves, 12 steps):

$$\frac{5 e + 8 g - \left(2 e + 5 g\right) x^2}{36 \left(x^4 - 5 x^2 + 4\right)^2} + \frac{x \left(17 d + 20 f + 32 h - \left(5 d + 8 f + 20 h\right) x^2\right)}{144 \left(x^4 - 5 x^2 + 4\right)^2} - \frac{\left(2 e + 5 g\right) \left(-2 x^2 + 5\right)}{108 \left(x^4 - 5 x^2 + 4\right)} - \frac{x \left(59 d + 380 f + 848 h - 5 \left(7 d + 28 f + 64 h\right) x^2\right)}{3456 \left(x^4 - 5 x^2 + 4\right)}$$
$$- \frac{\left(313 d + 820 f + 1936 h\right) \operatorname{arctanh}\left(\frac{x}{2}\right)}{20736} + \frac{\left(13 d + 25 f + 61 h\right) \operatorname{arctanh}(x)}{648} - \frac{\left(2 e + 5 g\right) \ln\left(-x^2 + 1\right)}{162} + \frac{\left(2 e + 5 g\right) \ln\left(-x^2 + 4\right)}{162}$$

Result(type 3, 461 leaves):

$$-\frac{d}{3456 \left(x - 2\right)^2} - \frac{e}{1728 \left(x - 2\right)^2} - \frac{f}{864 \left(x - 2\right)^2} - \frac{g}{432 \left(x - 2\right)^2} - \frac{h}{216 \left(x - 2\right)^2} + \frac{d}{3456 \left(2 + x\right)^2} - \frac{e}{1728 \left(2 + x\right)^2} + \frac{f}{864 \left(2 + x\right)^2} - \frac{g}{432 \left(2 + x\right)^2}$$
$$+ \frac{h}{216 \left(2 + x\right)^2} + \frac{d}{432 \left(x - 1\right)^2} + \frac{e}{432 \left(x - 1\right)^2} + \frac{f}{432 \left(x - 1\right)^2} + \frac{g}{432 \left(x - 1\right)^2} + \frac{h}{432 \left(x - 1\right)^2} - \frac{h}{432 \left(x + 1\right)^2} + \frac{g}{432 \left(x + 1\right)^2}$$
$$- \frac{f}{432 \left(x + 1\right)^2} + \frac{e}{432 \left(x + 1\right)^2} - \frac{d}{432 \left(x + 1\right)^2} + \frac{313 \ln(x - 2)\, d}{41472} + \frac{\ln(x - 2)\, e}{81} + \frac{\ln(2 + x)\, e}{81} - \frac{313 \ln(2 + x)\, d}{41472} - \frac{13 \ln(x - 1)\, d}{1296}$$
$$- \frac{\ln(x - 1)\, e}{81} + \frac{13 \ln(x + 1)\, d}{1296} - \frac{\ln(x + 1)\, e}{81} + \frac{25 \ln(x + 1)\, f}{1296} + \frac{h}{48 \left(x + 1\right)} - \frac{7 g}{432 \left(x + 1\right)} + \frac{5 f}{432 \left(x + 1\right)} - \frac{e}{144 \left(x + 1\right)} + \frac{d}{432 \left(x + 1\right)}$$
$$+ \frac{205 \ln(x - 2)\, f}{10368} + \frac{5 \ln(x - 2)\, g}{162} + \frac{121 \ln(x - 2)\, h}{2592} + \frac{19 d}{6912 \left(x - 2\right)} + \frac{17 e}{3456 \left(x - 2\right)} + \frac{5 f}{576 \left(x - 2\right)} + \frac{13 g}{864 \left(x - 2\right)} + \frac{11 h}{432 \left(x - 2\right)}$$
$$+ \frac{19 d}{6912 \left(2 + x\right)} - \frac{17 e}{3456 \left(2 + x\right)} + \frac{5 f}{576 \left(2 + x\right)} - \frac{13 g}{864 \left(2 + x\right)} + \frac{11 h}{432 \left(2 + x\right)} - \frac{205 \ln(2 + x)\, f}{10368} + \frac{5 \ln(2 + x)\, g}{162} - \frac{121 \ln(2 + x)\, h}{2592}$$
$$+ \frac{d}{432 \left(x - 1\right)} + \frac{e}{144 \left(x - 1\right)} + \frac{5 f}{432 \left(x - 1\right)} + \frac{7 g}{432 \left(x - 1\right)} + \frac{h}{48 \left(x - 1\right)} - \frac{25 \ln(x - 1)\, f}{1296} - \frac{5 \ln(x - 1)\, g}{162} - \frac{61 \ln(x - 1)\, h}{1296} + \frac{61 \ln(x + 1)\, h}{1296}$$
$$- \frac{5 \ln(x + 1)\, g}{162}$$

Problem 18: Humongous result has more than 20000 leaves.

$$\int \frac{i x^5 + h x^4 + g x^3 + f x^2 + e x + d}{\left(c x^4 + b x^2 + a\right)^3}\, dx$$

Optimal(type 3, 676 leaves, 12 steps):

$$\frac{x \left(b^2 d - a b f - 2 a \left(-a h + c d\right) + \left(a b h - 2 a c f + b c d\right) x^2\right)}{4 a \left(-4 a c + b^2\right) \left(c x^4 + b x^2 + a\right)^2} + \frac{2 a c g - b \left(a i + c e\right) - \left(-2 a c i + b^2 i - b c g + 2 c^2 e\right) x^2}{4 c \left(-4 a c + b^2\right) \left(c x^4 + b x^2 + a\right)^2}$$

$$+\frac{\left(6ce-3bg+2ai+\frac{b^2 i}{c}\right)\left(2cx^2+b\right)}{4\left(-4ac+b^2\right)^2\left(cx^4+bx^2+a\right)}$$

$$+\frac{x\left(3b^4 d+ab^3 f+8a^2 bcf+4a^2 c\left(ah+7cd\right)-ab^2\left(7ah+25cd\right)+c\left(3b^3 d+ab^2 f+20a^2 cf-12ab\left(ah+2cd\right)\right)x^2\right)}{8a^2\left(-4ac+b^2\right)^2\left(cx^4+bx^2+a\right)}$$

$$-\frac{\left(2aci+b^2 i-3bcg+6c^2 e\right)\operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{\left(-4ac+b^2\right)^{5/2}}$$

$$+\frac{1}{16a^2\left(-4ac+b^2\right)^2\sqrt{b-\sqrt{-4ac+b^2}}}\left(\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\sqrt{c}\left(3b^3 d+ab^2 f+20a^2 cf-12ab\left(ah+2cd\right)\right.\right.$$
$$\left.\left.+\frac{3b^4 d+ab^3 f-52a^2 bcf-6ab^2\left(-3ah+5cd\right)+24a^2 c\left(ah+7cd\right)}{\sqrt{-4ac+b^2}}\right)\sqrt{2}\right)$$

$$+\frac{1}{16a^2\left(-4ac+b^2\right)^2\sqrt{b+\sqrt{-4ac+b^2}}}\left(\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\sqrt{c}\left(3b^3 d+ab^2 f+20a^2 cf-12ab\left(ah+2cd\right)\right.\right.$$
$$\left.\left.+\frac{-3b^4 d-ab^3 f+52a^2 bcf+6ab^2\left(-3ah+5cd\right)-24a^2 c\left(ah+7cd\right)}{\sqrt{-4ac+b^2}}\right)\sqrt{2}\right)$$

Result(type ?, 21160 leaves): Display of huge result suppressed!

Problem 19: Humongous result has more than 20000 leaves.

$$\int \frac{kx^{11}+jx^8+ix^5+hx^4+gx^3+fx^2+ex+d}{\left(cx^4+bx^2+a\right)^3}\,dx$$

Optimal(type 3, 1123 leaves, 13 steps):

$$-\frac{x\left(c^2\left(abf-b^2\left(d+\frac{a^2 j}{c^2}\right)+2a\left(cd-ah+\frac{a^2 j}{c}\right)\right)+\left(2ac^3 f-ab^3 j-bc\left(-3a^2 j+ach+c^2 d\right)\right)x^2\right)}{4ac^2\left(-4ac+b^2\right)\left(cx^4+bx^2+a\right)^2}$$

$$+\frac{-bc^3\left(ai+ce\right)+ab^4 k-4a^2 b^2 ck+2ac^2\left(a^2 k+c^2 g\right)-\left(2c^5 e+b^2 c^3 i-c^4\left(2ai+bg\right)-b^5 k+5ab^3 ck-5a^2 bc^2 k\right)x^2}{4c^4\left(-4ac+b^2\right)\left(cx^4+bx^2+a\right)^2}$$

$$+\frac{1}{8a^2 c\left(-4ac+b^2\right)^2\left(cx^4+bx^2+a\right)}\left(x\left(c\left(ab^3 f+8a^2 bcf+4a^2\left(-9a^2 j+ach+7c^2 d\right)+b^4\left(3d-\frac{2a^2 j}{c^2}\right)-ab^2\left(25cd+7ah-\frac{11a^2 j}{c}\right)\right)\right.\right.$$
$$\left.\left.+\left(ab^2 c^2 f+20a^2 c^3 f+b^3\left(a^2 j+3c^2 d\right)-4abc\left(4a^2 j+3ach+6c^2 d\right)\right)x^2\right)\right)+\frac{1}{4c^3\left(-4ac+b^2\right)^2\left(cx^4+bx^2+a\right)}\left(b^3 c^2 i+2bc^3\left(ai\right.\right.$$
$$\left.\left.+3ce\right)+11ab^4 k-\frac{b^6 k}{c}+32a^3 c^2 k-3b^2\left(13a^2 ck+c^3 g\right)+2\left(6c^5 e+b^2 c^3 i-c^4\left(-2ai+3bg\right)+2b^5 k-15ab^3 ck+25a^2 bc^2 k\right)x^2\right)$$

$$-\frac{\left(12 c^5 e+2 b^2 c^3 i-c^4 (-4 a i+6 b g)-b^5 k+10 a b^3 c k-30 a^2 b c^2 k\right) \operatorname{arctanh}\left(\frac{2 c x^2+b}{\sqrt{-4 a c+b^2}}\right)}{2 c^3 (-4 a c+b^2)^{5/2}}+\frac{k \ln(c x^4+b x^2+a)}{4 c^3}$$

$$+\frac{1}{16 a^2 c^{3/2} (-4 a c+b^2)^2 \sqrt{b-\sqrt{-4 a c+b^2}}}\left(\arctan\left(\frac{x \sqrt{2} \sqrt{c}}{\sqrt{b-\sqrt{-4 a c+b^2}}}\right)\left(a b^2 c^2 f+20 a^2 c^3 f+b^3 (a^2 j+3 c^2 d)-4 a b c (4 a^2 j+3 a c h+6 c^2 d)+\frac{a b^3 c^2 f-52 a^2 b c^3 f-6 a b^2 c (-3 a^2 j-3 a c h+5 c^2 d)+b^4 (-a^2 j+3 c^2 d)+8 a^2 c^2 (5 a^2 j+3 a c h+21 c^2 d)}{\sqrt{-4 a c+b^2}}\right)\sqrt{2}\right)$$

$$+\frac{1}{16 a^2 c^{3/2} (-4 a c+b^2)^2 \sqrt{b+\sqrt{-4 a c+b^2}}}\left(\arctan\left(\frac{x \sqrt{2} \sqrt{c}}{\sqrt{b+\sqrt{-4 a c+b^2}}}\right)\left(a b^2 c^2 f+20 a^2 c^3 f+b^3 (a^2 j+3 c^2 d)-4 a b c (4 a^2 j+3 a c h+6 c^2 d)+\frac{-a b^3 c^2 f+52 a^2 b c^3 f+6 a b^2 c (-3 a^2 j-3 a c h+5 c^2 d)-b^4 (-a^2 j+3 c^2 d)-8 a^2 c^2 (5 a^2 j+3 a c h+21 c^2 d)}{\sqrt{-4 a c+b^2}}\right)\sqrt{2}\right)$$

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Result(type ?, 35335 leaves): Display of huge result suppressed!
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Problem 20: Result more than twice size of optimal antiderivative.
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$$\int \frac{a d+a e x+(a f+b d) x^2+b e x^3+(b f+c d) x^4+c e x^5+c f x^6}{(c x^4+b x^2+a)^2}\, dx$$

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Optimal(type 3, 171 leaves, 9 steps):
```

$$-\frac{e \operatorname{arctanh}\left(\frac{2 c x^2+b}{\sqrt{-4 a c+b^2}}\right)}{\sqrt{-4 a c+b^2}}+\frac{\arctan\left(\frac{x \sqrt{2} \sqrt{c}}{\sqrt{b-\sqrt{-4 a c+b^2}}}\right)\left(f+\frac{-b f+2 c d}{\sqrt{-4 a c+b^2}}\right)\sqrt{2}}{2 \sqrt{c} \sqrt{b-\sqrt{-4 a c+b^2}}}+\frac{\arctan\left(\frac{x \sqrt{2} \sqrt{c}}{\sqrt{b+\sqrt{-4 a c+b^2}}}\right)\left(f+\frac{b f-2 c d}{\sqrt{-4 a c+b^2}}\right)\sqrt{2}}{2 \sqrt{c} \sqrt{b+\sqrt{-4 a c+b^2}}}$$

```
Result(type 3, 615 leaves):
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$$-\frac{\sqrt{-4 a c+b^2} \ln\left(-2 c x^2+\sqrt{-4 a c+b^2}-b\right) e}{2 (4 a c-b^2)}-\frac{2 c \sqrt{2} \operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) f a}{(4 a c-b^2) \sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}+\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) f b^2}{2 (4 a c-b^2) \sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}$$

$$-\frac{\sqrt{-4 a c+b^2} \sqrt{2} \operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) b f}{2 (4 a c-b^2) \sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}+\frac{\sqrt{-4 a c+b^2}\, c \sqrt{2} \operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) d}{(4 a c-b^2) \sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}$$

$$+\frac{\sqrt{-4ac+b^2}\,\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)e}{2\left(4ac-b^2\right)}+\frac{2c\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)fa}{\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)fb^2}{2\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

$$-\frac{\sqrt{-4ac+b^2}\,\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)bf}{2\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{-4ac+b^2}\,c\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)d}{\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int\frac{ad+aex+(af+bd)x^2+bex^3+(bf+cd)x^4+cex^5+cfx^6}{\left(cx^4+bx^2+a\right)^3}\,dx$$

Optimal(type 3, 320 leaves, 11 steps):

$$-\frac{e\left(2cx^2+b\right)}{2\left(-4ac+b^2\right)\left(cx^4+bx^2+a\right)}+\frac{x\left(b^2d-2adc-abf+c(-2af+bd)x^2\right)}{2a\left(-4ac+b^2\right)\left(cx^4+bx^2+a\right)}+\frac{2ce\,\operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{\left(-4ac+b^2\right)^{3/2}}$$

$$+\frac{\arctan\left(\frac{x\sqrt{2}\,\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\sqrt{c}\left(bd-2af+\frac{4abf-12adc+b^2d}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4a\left(-4ac+b^2\right)\sqrt{b-\sqrt{-4ac+b^2}}}$$

$$+\frac{\arctan\left(\frac{x\sqrt{2}\,\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\sqrt{c}\left(bd-2af+\frac{-4abf+12adc-b^2d}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4a\left(-4ac+b^2\right)\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type ?, 2850 leaves): Display of huge result suppressed!

Problem 22: Result more than twice size of optimal antiderivative.

$$\int\frac{ad+aex+(af+bd)x^2+bex^3+(bf+cd)x^4+cex^5+cfx^6}{\left(cx^4+bx^2+a\right)^4}\,dx$$

Optimal(type 3, 561 leaves, 13 steps):

$$-\frac{e\left(2cx^2+b\right)}{4\left(-4ac+b^2\right)\left(cx^4+bx^2+a\right)^2}+\frac{x\left(b^2d-2adc-abf+c(-2af+bd)x^2\right)}{4a\left(-4ac+b^2\right)\left(cx^4+bx^2+a\right)^2}+\frac{3ce\left(2cx^2+b\right)}{2\left(-4ac+b^2\right)^2\left(cx^4+bx^2+a\right)}$$

$$+\frac{x\left(3b^4d-25ab^2cd+28a^2c^2d+ab^3f+8a^2bcf+c\left(20a^2cf+ab^2f-24abcd+3b^3d\right)x^2\right)}{8a^2\left(-4ac+b^2\right)^2\left(cx^4+bx^2+a\right)}-\frac{6c^2e\,\mathrm{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{\left(-4ac+b^2\right)^{5/2}}$$

$$+\frac{1}{16a^2\left(-4ac+b^2\right)^{5/2}\sqrt{b-\sqrt{-4ac+b^2}}}\left(\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\sqrt{c}\left(3b^4d+b^3\left(af+3d\sqrt{-4ac+b^2}\right)-4abc\left(13af+6d\sqrt{-4ac+b^2}\right)-ab^2\left(30cd-f\sqrt{-4ac+b^2}\right)+4a^2c\left(42cd+5f\sqrt{-4ac+b^2}\right)\right)\sqrt{2}\right)$$

$$+\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\sqrt{c}\left(3b^3d-24abcd+ab^2f+20a^2cf+\frac{52a^2bcf-168a^2c^2d-ab^3f+30ab^2cd-3b^4d}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{16a^2\left(-4ac+b^2\right)^2\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type ?, 10808 leaves): Display of huge result suppressed!

Test results for the 41 problems in "1.2.2.6 P(x) (d x)^m (a+b x^2+c x^4)^p.txt"

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2\left(Cx^2+Bx+A\right)}{cx^4+bx^2+a}\,dx$$

Optimal(type 3, 227 leaves, 11 steps):

$$\frac{Cx}{c}+\frac{B\ln\left(cx^4+bx^2+a\right)}{4c}+\frac{bB\,\mathrm{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{2c\sqrt{-4ac+b^2}}+\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\left(Ac-bC+\frac{-Abc+\left(-2ac+b^2\right)C}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{2c^{3/2}\sqrt{b-\sqrt{-4ac+b^2}}}$$

$$+\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(Ac-bC+\frac{Abc+2acC-b^2C}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{2c^{3/2}\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type 3, 1326 leaves):

$$\frac{Cx}{c}+\frac{B\ln\left(-2cx^2+\sqrt{-4ac+b^2}-b\right)b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{B\ln\left(-2cx^2+\sqrt{-4ac+b^2}-b\right)a}{4ac-b^2}-\frac{B\ln\left(-2cx^2+\sqrt{-4ac+b^2}-b\right)b^2}{4c\left(4ac-b^2\right)}$$

$$-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)Ab\sqrt{-4ac+b^2}}{2\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{2c\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)Aa}{\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

$$
+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) A b^2}{2\left(4 a c-b^2\right) \sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) C\left(-4 a c+b^2\right) b}{4 c\left(4 a c-b^2\right) \sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}
$$

$$
-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) C \sqrt{-4 a c+b^2}\, a}{\left(4 a c-b^2\right) \sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) C \sqrt{-4 a c+b^2}\, b^2}{2 c\left(4 a c-b^2\right) \sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}
$$

$$
+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) b C a}{\left(4 a c-b^2\right) \sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x \sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) b^3 C}{4 c\left(4 a c-b^2\right) \sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}
$$

$$
-\frac{B \ln\left(2 c x^2+\sqrt{-4 a c+b^2}+b\right) b \sqrt{-4 a c+b^2}}{4 c\left(4 a c-b^2\right)}+\frac{B \ln\left(2 c x^2+\sqrt{-4 a c+b^2}+b\right) a}{4 a c-b^2}-\frac{B \ln\left(2 c x^2+\sqrt{-4 a c+b^2}+b\right) b^2}{4 c\left(4 a c-b^2\right)}
$$

$$
-\frac{\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) A b \sqrt{-4 a c+b^2}}{2\left(4 a c-b^2\right) \sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}+\frac{2 c \sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) A a}{\left(4 a c-b^2\right) \sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}
$$

$$
-\frac{\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) A b^2}{2\left(4 a c-b^2\right) \sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}+\frac{\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) C\left(-4 a c+b^2\right) b}{4 c\left(4 a c-b^2\right) \sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}
$$

$$
-\frac{\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) C \sqrt{-4 a c+b^2}\, a}{\left(4 a c-b^2\right) \sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}+\frac{\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) C \sqrt{-4 a c+b^2}\, b^2}{2 c\left(4 a c-b^2\right) \sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}
$$

$$
-\frac{\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) b C a}{\left(4 a c-b^2\right) \sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}+\frac{\sqrt{2}\,\arctan\left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) b^3 C}{4 c\left(4 a c-b^2\right) \sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}
$$

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Problem 11: Result more than twice size of optimal antiderivative.
```

$$
\int \frac{x^4\left(C x^2+B x+A\right)}{\left(c x^4+b x^2+a\right)^2}\, dx
$$

Optimal(type 3, 363 leaves, 11 steps):

$$\frac{(2Ac-bC)x}{2c(-4ac+b^2)}+\frac{Bx^2(bx^2+2a)}{2(-4ac+b^2)(cx^4+bx^2+a)}-\frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2(-4ac+b^2)(cx^4+bx^2+a)}+\frac{2aB\operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{3/2}}$$
$$+\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\left(Abc+(-6ac+b^2)C+\frac{-Ac(4ac+b^2)-b(-8ac+b^2)C}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4c^{3/2}(-4ac+b^2)\sqrt{b-\sqrt{-4ac+b^2}}}$$
$$+\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(Abc+(-6ac+b^2)C+\frac{Ac(4ac+b^2)+b(-8ac+b^2)C}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4c^{3/2}(-4ac+b^2)\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type ?, 5282 leaves): Display of huge result suppressed!

Problem 12: Result more than twice size of optimal antiderivative.

$$\int\frac{Cx^2+Bx+A}{x^2(cx^4+bx^2+a)^2}\,dx$$

Optimal(type 3, 452 leaves, 15 steps):

$$\frac{10aAc-3Ab^2+abC}{2a^2(-4ac+b^2)x}+\frac{B(cx^2b-2ac+b^2)}{2a(-4ac+b^2)(cx^4+bx^2+a)}+\frac{A(-2ac+b^2)-abC+c(Ab-2aC)x^2}{2a(-4ac+b^2)x(cx^4+bx^2+a)}+\frac{bB(-6ac+b^2)\operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{2a^2(-4ac+b^2)^{3/2}}$$
$$+\frac{\ln(x)B}{a^2}-\frac{B\ln(cx^4+bx^2+a)}{4a^2}$$
$$-\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\sqrt{c}\left(-aC\left(b^2-12ac+b\sqrt{-4ac+b^2}\right)+A\left(3b^3-16abc+3b^2\sqrt{-4ac+b^2}-10ac\sqrt{-4ac+b^2}\right)\right)\sqrt{2}}{4a^2(-4ac+b^2)^{3/2}\sqrt{b-\sqrt{-4ac+b^2}}}$$
$$-\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\sqrt{c}\left(3Ab^2-10aAc-abC+\frac{-A(-16abc+3b^3)+a(-12ac+b^2)C}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4a^2(-4ac+b^2)\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type ?, 6476 leaves): Display of huge result suppressed!

Problem 13: Result more than twice size of optimal antiderivative.

$$\int(dx)^m(Cx^2+Bx+A)(cx^4+bx^2+a)\,dx$$

Optimal(type 3, 137 leaves, 2 steps):

$$\frac{aA(dx)^{1+m}}{d(1+m)}+\frac{aB(dx)^{2+m}}{d^2(2+m)}+\frac{(Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{bB(dx)^{4+m}}{d^4(4+m)}+\frac{(Ac+bC)(dx)^{5+m}}{d^5(5+m)}+\frac{Bc(dx)^{6+m}}{d^6(6+m)}+\frac{cC(dx)^{7+m}}{d^7(7+m)}$$

Result(type 3, 584 leaves):

$$\frac{1}{(7+m)(6+m)(5+m)(4+m)(3+m)(2+m)(1+m)}\Big(x\big(Ccm^6x^6+Bcm^6x^5+21Ccm^5x^6+Acm^6x^4+22Bcm^5x^5+Cbm^6x^4+175Ccm^4x^6$$
$$+23Acm^5x^4+Bbm^6x^3+190Bcm^4x^5+23Cbm^5x^4+735Ccm^3x^6+Abm^6x^2+207Acm^4x^4+24Bbm^5x^3+820Bcm^3x^5+Cam^6x^2+207Cbm^4x^4$$
$$+1624Ccm^2x^6+25Abm^5x^2+925Acm^3x^4+Bam^6x+226Bbm^4x^3+1849Bcm^2x^5+25Cam^5x^2+925Cbm^3x^4+1764Ccmx^6+Aam^6$$
$$+247Abm^4x^2+2144Acm^2x^4+26Bam^5x+1056Bbm^3x^3+2038Bcmx^5+247Cam^4x^2+2144Cbm^2x^4+720Ccx^6+27Aam^5+1219Abm^3x^2$$
$$+2412Acmx^4+270Bam^4x+2545Bbm^2x^3+840Bcx^5+1219Cam^3x^2+2412Cbmx^4+295Aam^4+3112Abm^2x^2+1008Acx^4+1420Bam^3x$$
$$+2952Bbmx^3+3112Cam^2x^2+1008Cbx^4+1665Aam^3+3796Abmx^2+3929Bam^2x+1260bBx^3+3796Camx^2+5104Aam^2+1680Abx^2$$
$$+5274Bamx+1680Cax^2+8028Aam+2520aBx+5040Aa\big)(dx)^m\Big)$$

Problem 14: Unable to integrate problem.

$$\int\frac{(dx)^m(Cx^2+Bx+A)}{(cx^4+bx^2+a)^2}\,dx$$

Optimal(type 5, 629 leaves, 10 steps):

$$\frac{B(dx)^{2+m}(cx^2b-2ac+b^2)}{2a(-4ac+b^2)d^2(cx^4+bx^2+a)}+\frac{(dx)^{1+m}(A(-2ac+b^2)-abC+c(Ab-2aC)x^2)}{2a(-4ac+b^2)d(cx^4+bx^2+a)}$$
$$+\frac{Bc(dx)^{2+m}\mathrm{hypergeom}\left(\left[1,1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right],-\frac{2x^2c}{b+\sqrt{-4ac+b^2}}\right)\left(4ac(2-m)+bm\left(b-\sqrt{-4ac+b^2}\right)\right)}{2a(-4ac+b^2)^{3/2}d^2(2+m)\left(b+\sqrt{-4ac+b^2}\right)}$$
$$-\frac{Bc(dx)^{2+m}\mathrm{hypergeom}\left(\left[1,1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right],-\frac{2x^2c}{b-\sqrt{-4ac+b^2}}\right)\left(4ac(2-m)+bm\left(b+\sqrt{-4ac+b^2}\right)\right)}{2a(-4ac+b^2)^{3/2}d^2(2+m)\left(b-\sqrt{-4ac+b^2}\right)}$$
$$-\frac{1}{2a(-4ac+b^2)^{3/2}d(1+m)\left(b+\sqrt{-4ac+b^2}\right)}\Bigg(c(dx)^{1+m}\mathrm{hypergeom}\left(\left[1,\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{2x^2c}{b+\sqrt{-4ac+b^2}}\right)\Big(2aC\Big(2b+(1$$
$$-m)\sqrt{-4ac+b^2}\Big)+A\left(b^2(1-m)-4ac(3-m)-b(1-m)\sqrt{-4ac+b^2}\right)\Big)\Bigg)$$
$$+\frac{1}{2a(-4ac+b^2)^{3/2}d(1+m)\left(b-\sqrt{-4ac+b^2}\right)}\Bigg(c(dx)^{1+m}\mathrm{hypergeom}\left(\left[1,\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{2x^2c}{b-\sqrt{-4ac+b^2}}\right)\Big(2aC\Big(2b-(1$$
$$-m)\sqrt{-4ac+b^2}\Big)+A\left(b^2(1-m)-4ac(3-m)+b(1-m)\sqrt{-4ac+b^2}\right)\Big)\Bigg)$$

Result(type 8, 32 leaves):

$$\int \frac{(d x)^m (C x^2 + B x + A)}{(c x^4 + b x^2 + a)^2}\, dx$$

Problem 15: Result more than twice size of optimal antiderivative.

$$\int \frac{C x^5 + B x^4 + A x^3}{x (c x^4 + b x^2 + a)^2}\, dx$$

Optimal(type 3, 306 leaves, 11 steps):

$$\frac{B (b x^2 + 2 a)}{2 (-4 a c + b^2) (c x^4 + b x^2 + a)} - \frac{x (A b - 2 a C + (2 A c - b C) x^2)}{2 (-4 a c + b^2) (c x^4 + b x^2 + a)} - \frac{b B \operatorname{arctanh}\left(\frac{2 c x^2 + b}{\sqrt{-4 a c + b^2}}\right)}{(-4 a c + b^2)^{3/2}}$$
$$- \frac{\arctan\left(\frac{x \sqrt{2} \sqrt{c}}{\sqrt{b - \sqrt{-4 a c + b^2}}}\right) \left(2 A c - b C + \frac{-4 A b c + (4 a c + b^2) C}{\sqrt{-4 a c + b^2}}\right) \sqrt{2}}{4 (-4 a c + b^2) \sqrt{c} \sqrt{b - \sqrt{-4 a c + b^2}}}$$
$$- \frac{\arctan\left(\frac{x \sqrt{2} \sqrt{c}}{\sqrt{b + \sqrt{-4 a c + b^2}}}\right) \left(2 A c - b C + \frac{4 A b c - (4 a c + b^2) C}{\sqrt{-4 a c + b^2}}\right) \sqrt{2}}{4 (-4 a c + b^2) \sqrt{c} \sqrt{b + \sqrt{-4 a c + b^2}}}$$

Result(type ?, 4062 leaves): Display of huge result suppressed!

Problem 16: Result more than twice size of optimal antiderivative.

$$\int \frac{x^7 (f x^4 + e x^2 + d)}{c x^4 + b x^2 + a}\, dx$$

Optimal(type 3, 257 leaves, 7 steps):

$$\frac{(b^2 c e - a c^2 e - b^3 f - b c (-2 a f + c d)) x^2}{2 c^4} + \frac{(c^2 d + b^2 f - c (a f + b e)) x^4}{4 c^3} + \frac{(-b f + c e) x^6}{6 c^2} + \frac{f x^8}{8 c}$$
$$- \frac{(b^3 c e - 2 a b c^2 e - b^4 f - b^2 c (-3 a f + c d) + a c^2 (-a f + c d)) \ln(c x^4 + b x^2 + a)}{4 c^5}$$
$$- \frac{(b^4 c e - 4 a b^2 c^2 e + 2 a^2 c^3 e - b^5 f - b^3 c (-5 a f + c d) + a b c^2 (-5 a f + 3 c d)) \operatorname{arctanh}\left(\frac{2 c x^2 + b}{\sqrt{-4 a c + b^2}}\right)}{2 c^5 \sqrt{-4 a c + b^2}}$$

Result(type 3, 621 leaves):

$$\frac{fx^8}{8c}+\frac{abfx^2}{c^3}-\frac{3\ln(cx^4+bx^2+a)ab^2f}{4c^4}+\frac{x^6e}{6c}+\frac{x^4d}{4c}+\frac{\ln(cx^4+bx^2+a)abe}{2c^3}+\frac{\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)a^2e}{c^2\sqrt{4ac-b^2}}-\frac{\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)b^5f}{2c^5\sqrt{4ac-b^2}}$$
$$+\frac{\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)b^4e}{2c^4\sqrt{4ac-b^2}}-\frac{\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)b^3d}{2c^3\sqrt{4ac-b^2}}-\frac{x^6bf}{6c^2}-\frac{x^4af}{4c^2}+\frac{x^4b^2f}{4c^3}-\frac{x^4be}{4c^2}-\frac{x^2ae}{2c^2}-\frac{b^3fx^2}{2c^4}+\frac{x^2b^2e}{2c^3}-\frac{bdx^2}{2c^2}$$
$$+\frac{\ln(cx^4+bx^2+a)a^2f}{4c^3}-\frac{\ln(cx^4+bx^2+a)ad}{4c^2}+\frac{\ln(cx^4+bx^2+a)b^4f}{4c^5}-\frac{\ln(cx^4+bx^2+a)b^3e}{4c^4}+\frac{\ln(cx^4+bx^2+a)b^2d}{4c^3}$$
$$-\frac{5\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)a^2bf}{2c^3\sqrt{4ac-b^2}}+\frac{5\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)ab^3f}{2c^4\sqrt{4ac-b^2}}-\frac{2\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)ab^2e}{c^3\sqrt{4ac-b^2}}+\frac{3\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)abd}{2c^2\sqrt{4ac-b^2}}$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int\frac{x^3(fx^4+ex^2+d)}{cx^4+bx^2+a}\,dx$$

Optimal(type 3, 132 leaves, 7 steps):

$$\frac{(-bf+ce)x^2}{2c^2}+\frac{fx^4}{4c}+\frac{(c^2d+b^2f-c(af+be))\ln(cx^4+bx^2+a)}{4c^3}-\frac{(b^2ce-2ac^2e-b^3f-bc(-3af+cd))\operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{2c^3\sqrt{-4ac+b^2}}$$

Result(type 3, 320 leaves):

$$\frac{fx^4}{4c}-\frac{x^2bf}{2c^2}+\frac{x^2e}{2c}-\frac{\ln(cx^4+bx^2+a)af}{4c^2}+\frac{\ln(cx^4+bx^2+a)b^2f}{4c^3}-\frac{\ln(cx^4+bx^2+a)be}{4c^2}+\frac{\ln(cx^4+bx^2+a)d}{4c}+\frac{3\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)abf}{2c^2\sqrt{4ac-b^2}}$$
$$-\frac{\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)ae}{c\sqrt{4ac-b^2}}-\frac{\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)b^3f}{2c^3\sqrt{4ac-b^2}}+\frac{\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)b^2e}{2c^2\sqrt{4ac-b^2}}-\frac{\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)bd}{2c\sqrt{4ac-b^2}}$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int\frac{x^2(fx^4+ex^2+d)}{cx^4+bx^2+a}\,dx$$

Optimal(type 3, 244 leaves, 5 steps):

$$\frac{(-bf+ce)x}{c^2}+\frac{fx^3}{3c}+\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\left(c^2d-bce+b^2f-acf+\frac{b^2ce-2ac^2e-b^3f-bc(-3af+cd)}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{2c^{5/2}\sqrt{b-\sqrt{-4ac+b^2}}}$$

$$+\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(c^2d-bce+b^2f-acf+\frac{-b^2ce+2ac^2e+b^3f+bc(-3af+cd)}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{2c^{5/2}\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type 3, 1034 leaves):

$$\frac{fx^3}{3c}-\frac{bfx}{c^2}+\frac{ex}{c}+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)af}{2c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)b^2f}{2c^2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

$$+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)be}{2c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)d}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{3\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)abf}{2c\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

$$+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)ae}{\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)b^3f}{2c^2\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)b^2e}{2c\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

$$+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)bd}{2\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)af}{2c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)b^2f}{2c^2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

$$-\frac{\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)be}{2c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)d}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{3\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)abf}{2c\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

$$+\frac{\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)ae}{\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)b^3f}{2c^2\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)b^2e}{2c\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

$$+\frac{\sqrt{2}\arctan\left(\frac{c x\sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) b d}{2\sqrt{-4 a c+b^2}\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}$$

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Problem 19: Result more than twice size of optimal antiderivative.
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$$\int \frac{f x^4+e x^2+d}{x^4\left(c x^4+b x^2+a\right)}\,dx$$

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Optimal(type 3, 226 leaves, 5 steps):
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$$-\frac{d}{3 a x^3}+\frac{-a e+b d}{x a^2}+\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4 a c+b^2}}}\right)\sqrt{c}\left(b d-a e+\frac{b^2 d-e a b-2 a\left(-a f+c d\right)}{\sqrt{-4 a c+b^2}}\right)\sqrt{2}}{2 a^2\sqrt{b-\sqrt{-4 a c+b^2}}}$$

$$-\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4 a c+b^2}}}\right)\sqrt{c}\left(b^2 d-b\left(a e+d\sqrt{-4 a c+b^2}\right)-a\left(2 c d-2 a f-e\sqrt{-4 a c+b^2}\right)\right)\sqrt{2}}{2 a^2\sqrt{-4 a c+b^2}\sqrt{b+\sqrt{-4 a c+b^2}}}$$

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Result(type 3, 726 leaves):
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$$\frac{c\sqrt{2}\,\mathrm{arctanh}\left(\frac{c x\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) e}{2 a\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}-\frac{c\sqrt{2}\,\mathrm{arctanh}\left(\frac{c x\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) b d}{2 a^2\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}-\frac{c\sqrt{2}\,\mathrm{arctanh}\left(\frac{c x\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) f}{\sqrt{-4 a c+b^2}\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}$$

$$+\frac{c\sqrt{2}\,\mathrm{arctanh}\left(\frac{c x\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) e b}{2 a\sqrt{-4 a c+b^2}\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}+\frac{c^2\sqrt{2}\,\mathrm{arctanh}\left(\frac{c x\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) d}{a\sqrt{-4 a c+b^2}\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}-\frac{c\sqrt{2}\,\mathrm{arctanh}\left(\frac{c x\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right) b^2 d}{2 a^2\sqrt{-4 a c+b^2}\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}$$

$$-\frac{c\sqrt{2}\arctan\left(\frac{c x\sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) e}{2 a\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}+\frac{c\sqrt{2}\arctan\left(\frac{c x\sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) b d}{2 a^2\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}-\frac{c\sqrt{2}\arctan\left(\frac{c x\sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) f}{\sqrt{-4 a c+b^2}\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}$$

$$+\frac{c\sqrt{2}\arctan\left(\frac{c x\sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) e b}{2 a\sqrt{-4 a c+b^2}\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}+\frac{c^2\sqrt{2}\arctan\left(\frac{c x\sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) d}{a\sqrt{-4 a c+b^2}\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}-\frac{c\sqrt{2}\arctan\left(\frac{c x\sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right) b^2 d}{2 a^2\sqrt{-4 a c+b^2}\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}$$

$$-\frac{d}{3\,a\,x^3}-\frac{e}{a\,x}+\frac{b\,d}{a^2\,x}$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int\frac{fx^4+ex^2+d}{x\left(cx^4+bx^2+a\right)^2}\,\mathrm{d}x$$

Optimal(type 3, 156 leaves, 8 steps):

$$\frac{b^2 d-e a b-2 a\left(-a f+c d\right)+\left(a b f-2 a c e+b c d\right) x^2}{2 a\left(-4 a c+b^2\right)\left(c x^4+b x^2+a\right)}+\frac{\left(b^3 d+4 a^2 c e-2 a b\left(a f+3 c d\right)\right)\operatorname{arctanh}\left(\frac{2 c x^2+b}{\sqrt{-4 a c+b^2}}\right)}{2 a^2\left(-4 a c+b^2\right)^{3/2}}+\frac{d\ln(x)}{a^2}-\frac{d\ln\left(c x^4+b x^2+a\right)}{4 a^2}$$

Result(type 3, 743 leaves):

$$-\frac{x^2 b f}{2\left(c x^4+b x^2+a\right)\left(4 a c-b^2\right)}+\frac{c x^2 e}{\left(c x^4+b x^2+a\right)\left(4 a c-b^2\right)}-\frac{x^2 b c d}{2 a\left(c x^4+b x^2+a\right)\left(4 a c-b^2\right)}-\frac{a f}{\left(c x^4+b x^2+a\right)\left(4 a c-b^2\right)}$$
$$+\frac{b e}{2\left(c x^4+b x^2+a\right)\left(4 a c-b^2\right)}+\frac{d c}{\left(c x^4+b x^2+a\right)\left(4 a c-b^2\right)}-\frac{b^2 d}{2 a\left(c x^4+b x^2+a\right)\left(4 a c-b^2\right)}-\frac{c\ln\left(\left(4 a c-b^2\right)\left(c x^4+b x^2+a\right)\right) d}{a\left(4 a c-b^2\right)}$$
$$+\frac{\ln\left(\left(4 a c-b^2\right)\left(c x^4+b x^2+a\right)\right) b^2 d}{4 a^2\left(4 a c-b^2\right)}-\frac{\arctan\left(\frac{2\left(4 a c-b^2\right) c x^2+\left(4 a c-b^2\right) b}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}\right) b f}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}$$
$$+\frac{2\arctan\left(\frac{2\left(4 a c-b^2\right) c x^2+\left(4 a c-b^2\right) b}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}\right) c e}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}-\frac{3\arctan\left(\frac{2\left(4 a c-b^2\right) c x^2+\left(4 a c-b^2\right) b}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}\right) b c d}{a\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}$$
$$+\frac{\arctan\left(\frac{2\left(4 a c-b^2\right) c x^2+\left(4 a c-b^2\right) b}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}\right) b^3 d}{2 a^2\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}+\frac{d\ln(x)}{a^2}$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int\frac{fx^4+ex^2+d}{x^3\left(cx^4+bx^2+a\right)^2}\,\mathrm{d}x$$

Optimal(type 3, 222 leaves, 8 steps):

$$-\frac{d}{2 a^2 x^2}+\frac{-b^3 d+a b^2 e-2 a^2 c e+a b\left(-a f+3 c d\right)-c\left(b^2 d-e a b-2 a\left(-a f+c d\right)\right) x^2}{2 a^2\left(-4 a c+b^2\right)\left(c x^4+b x^2+a\right)}$$

$$-\frac{\left(2 b^4 d-12 a b^2 c d-a b^3 e+6 a^2 b c e+4 a^2 c(-a f+3 c d)\right) \operatorname{arctanh}\left(\frac{2 c x^2+b}{\sqrt{-4 a c+b^2}}\right)}{2 a^3\left(-4 a c+b^2\right)^{3/2}}-\frac{(-a e+2 b d) \ln (x)}{a^3}+\frac{(-a e+2 b d) \ln \left(c x^4+b x^2+a\right)}{4 a^3}$$

Result(type 3, 1155 leaves):

$$\frac{c x^2 f}{\left(c x^4+b x^2+a\right)\left(4 a c-b^2\right)}-\frac{c x^2 e b}{2 a\left(c x^4+b x^2+a\right)\left(4 a c-b^2\right)}-\frac{c^2 x^2 d}{a\left(c x^4+b x^2+a\right)\left(4 a c-b^2\right)}+\frac{c x^2 b^2 d}{2 a^2\left(c x^4+b x^2+a\right)\left(4 a c-b^2\right)}$$
$$+\frac{b f}{2\left(c x^4+b x^2+a\right)\left(4 a c-b^2\right)}+\frac{c e}{\left(c x^4+b x^2+a\right)\left(4 a c-b^2\right)}-\frac{b^2 e}{2 a\left(c x^4+b x^2+a\right)\left(4 a c-b^2\right)}-\frac{3 b c d}{2 a\left(c x^4+b x^2+a\right)\left(4 a c-b^2\right)}$$
$$+\frac{b^3 d}{2 a^2\left(c x^4+b x^2+a\right)\left(4 a c-b^2\right)}-\frac{c \ln \left(\left(4 a c-b^2\right)\left(c x^4+b x^2+a\right)\right) e}{a\left(4 a c-b^2\right)}+\frac{\ln \left(\left(4 a c-b^2\right)\left(c x^4+b x^2+a\right)\right) b^2 e}{4 a^2\left(4 a c-b^2\right)}$$
$$+\frac{2 c \ln \left(\left(4 a c-b^2\right)\left(c x^4+b x^2+a\right)\right) b d}{a^2\left(4 a c-b^2\right)}-\frac{\ln \left(\left(4 a c-b^2\right)\left(c x^4+b x^2+a\right)\right) b^3 d}{2 a^3\left(4 a c-b^2\right)}+\frac{2 \arctan \left(\frac{2\left(4 a c-b^2\right) c x^2+\left(4 a c-b^2\right) b}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}\right) c f}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}$$
$$-\frac{3 \arctan \left(\frac{2\left(4 a c-b^2\right) c x^2+\left(4 a c-b^2\right) b}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}\right) b c e}{a \sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}-\frac{6 \arctan \left(\frac{2\left(4 a c-b^2\right) c x^2+\left(4 a c-b^2\right) b}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}\right) c^2 d}{a \sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}$$
$$+\frac{\arctan \left(\frac{2\left(4 a c-b^2\right) c x^2+\left(4 a c-b^2\right) b}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}\right) b^3 e}{2 a^2 \sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}+\frac{6 \arctan \left(\frac{2\left(4 a c-b^2\right) c x^2+\left(4 a c-b^2\right) b}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}\right) b^2 c d}{a^2 \sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}$$
$$-\frac{\arctan \left(\frac{2\left(4 a c-b^2\right) c x^2+\left(4 a c-b^2\right) b}{\sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}\right) b^4 d}{a^3 \sqrt{64 a^3 c^3-48 a^2 b^2 c^2+12 a b^4 c-b^6}}-\frac{d}{2 a^2 x^2}+\frac{\ln (x) e}{a^2}-\frac{2 \ln (x) b d}{a^3}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2\left(f x^4+e x^2+d\right)}{\left(c x^4+b x^2+a\right)^2} \, dx$$

Optimal(type 3, 321 leaves, 4 steps):

$$-\frac{x\left(b c d-2 a c e+a b f+\left(-2 a c f+b^2 f-b c e+2 c^2 d\right) x^2\right)}{2 c\left(-4 a c+b^2\right)\left(c x^4+b x^2+a\right)}$$
$$-\frac{\arctan \left(\frac{x \sqrt{2} \sqrt{c}}{\sqrt{b-\sqrt{-4 a c+b^2}}}\right)\left(2 c d-b e+6 a f-\frac{b^2 f}{c}+\frac{b^2 c e+4 a c^2 e+b^3 f-4 b c(2 a f+c d)}{c \sqrt{-4 a c+b^2}}\right) \sqrt{2}}{4\left(-4 a c+b^2\right) \sqrt{c} \sqrt{b-\sqrt{-4 a c+b^2}}}$$

$$-\frac{\arctan\left(\frac{x\sqrt{2}\,\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\left(2cd-be+6af-\frac{b^2 f}{c}+\frac{-b^2ce-4ac^2e-b^3f+4bc(2af+cd)}{c\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4\left(-4ac+b^2\right)\sqrt{c}\,\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type ?, 5527 leaves): Display of huge result suppressed!

Problem 31: Result more than twice size of optimal antiderivative.

$$\int \frac{5x^6+3x^4+x^2+4}{\left(x^4+2x^2+3\right)^2}\,dx$$

Optimal(type 3, 154 leaves, 10 steps):

$$\frac{25x\left(-x^2+1\right)}{24\left(x^4+2x^2+3\right)}-\frac{\arctan\left(\frac{-2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{-69402+77382\sqrt{3}}}{288}+\frac{\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{-69402+77382\sqrt{3}}}{288}$$
$$+\frac{\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{69402+77382\sqrt{3}}}{576}-\frac{\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{69402+77382\sqrt{3}}}{576}$$

Result(type 3, 407 leaves):

$$\frac{-\frac{25}{24}x^3+\frac{25}{24}x}{x^4+2x^2+3}-\frac{139\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\,\sqrt{3}}{576}-\frac{11\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{48}$$
$$+\frac{139\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)\sqrt{3}}{288\sqrt{2+2\sqrt{3}}}+\frac{11\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)}{24\sqrt{2+2\sqrt{3}}}+\frac{7\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{3}}{72\sqrt{2+2\sqrt{3}}}$$
$$+\frac{139\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\,\sqrt{3}}{576}+\frac{11\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{48}$$
$$+\frac{139\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)\sqrt{3}}{288\sqrt{2+2\sqrt{3}}}+\frac{11\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)}{24\sqrt{2+2\sqrt{3}}}+\frac{7\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{3}}{72\sqrt{2+2\sqrt{3}}}$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \frac{x^{10}\left(5x^6+3x^4+x^2+4\right)}{\left(x^4+2x^2+3\right)^3}\,dx$$

Optimal(type 3, 187 leaves, 13 steps):

$$58x-9x^3+x^5-\frac{25x(7x^2+15)}{16(x^4+2x^2+3)^2}+\frac{x(252x^2+3305)}{64(x^4+2x^2+3)}+\frac{3\arctan\left(\frac{-2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{-8595619+7678611\sqrt{3}}}{256}$$
$$-\frac{3\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{-8595619+7678611\sqrt{3}}}{256}+\frac{3\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{8595619+7678611\sqrt{3}}}{512}$$
$$-\frac{3\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{8595619+7678611\sqrt{3}}}{512}$$

Result(type 3, 428 leaves):

$$x^5-9x^3+58x+\frac{\frac{63}{16}x^7+\frac{3809}{64}x^5+\frac{3333}{32}x^3+\frac{8415}{64}x}{(x^4+2x^2+3)^2}-\frac{5091\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{1024}$$
$$-\frac{14385\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024}+\frac{5091\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}$$
$$+\frac{14385\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)}{512\sqrt{2+2\sqrt{3}}}-\frac{4647\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{3}}{64\sqrt{2+2\sqrt{3}}}$$
$$+\frac{5091\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{1024}+\frac{14385\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024}$$
$$+\frac{5091\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}+\frac{14385\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)}{512\sqrt{2+2\sqrt{3}}}$$
$$-\frac{4647\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{3}}{64\sqrt{2+2\sqrt{3}}}$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int \frac{x^8 \left(5 x^6 + 3 x^4 + x^2 + 4\right)}{\left(x^4 + 2 x^2 + 3\right)^3} \, dx$$

Optimal(type 3, 184 leaves, 13 steps):

$$-27 x + \frac{5 x^3}{3} + \frac{25 x \left(5 x^2 + 3\right)}{16 \left(x^4 + 2 x^2 + 3\right)^2} - \frac{x \left(835 x^2 + 1468\right)}{64 \left(x^4 + 2 x^2 + 3\right)} - \frac{21 \ln\left(x^2 + \sqrt{3} - x \sqrt{-2 + 2 \sqrt{3}}\right) \sqrt{-34271 + 22721 \sqrt{3}}}{512}$$
$$+ \frac{21 \ln\left(x^2 + \sqrt{3} + x \sqrt{-2 + 2 \sqrt{3}}\right) \sqrt{-34271 + 22721 \sqrt{3}}}{512} - \frac{21 \arctan\left(\frac{-2 x + \sqrt{-2 + 2 \sqrt{3}}}{\sqrt{2 + 2 \sqrt{3}}}\right) \sqrt{34271 + 22721 \sqrt{3}}}{256}$$
$$+ \frac{21 \arctan\left(\frac{2 x + \sqrt{-2 + 2 \sqrt{3}}}{\sqrt{2 + 2 \sqrt{3}}}\right) \sqrt{34271 + 22721 \sqrt{3}}}{256}$$

Result(type 3, 425 leaves):

$$\frac{5 x^3}{3} - 27 x + \frac{-\frac{835}{64} x^7 - \frac{1569}{32} x^5 - \frac{4941}{64} x^3 - \frac{513}{8} x}{\left(x^4 + 2 x^2 + 3\right)^2} - \frac{693 \ln\left(x^2 + \sqrt{3} + x \sqrt{-2 + 2 \sqrt{3}}\right) \sqrt{-2 + 2 \sqrt{3}} \sqrt{3}}{1024}$$
$$+ \frac{3675 \ln\left(x^2 + \sqrt{3} + x \sqrt{-2 + 2 \sqrt{3}}\right) \sqrt{-2 + 2 \sqrt{3}}}{1024} + \frac{693 \arctan\left(\frac{2 x + \sqrt{-2 + 2 \sqrt{3}}}{\sqrt{2 + 2 \sqrt{3}}}\right) \left(-2 + 2 \sqrt{3}\right) \sqrt{3}}{512 \sqrt{2 + 2 \sqrt{3}}}$$
$$- \frac{3675 \arctan\left(\frac{2 x + \sqrt{-2 + 2 \sqrt{3}}}{\sqrt{2 + 2 \sqrt{3}}}\right) \left(-2 + 2 \sqrt{3}\right)}{512 \sqrt{2 + 2 \sqrt{3}}} + \frac{273 \arctan\left(\frac{2 x + \sqrt{-2 + 2 \sqrt{3}}}{\sqrt{2 + 2 \sqrt{3}}}\right) \sqrt{3}}{8 \sqrt{2 + 2 \sqrt{3}}}$$
$$+ \frac{693 \ln\left(x^2 + \sqrt{3} - x \sqrt{-2 + 2 \sqrt{3}}\right) \sqrt{-2 + 2 \sqrt{3}} \sqrt{3}}{1024} - \frac{3675 \ln\left(x^2 + \sqrt{3} - x \sqrt{-2 + 2 \sqrt{3}}\right) \sqrt{-2 + 2 \sqrt{3}}}{1024}$$
$$+ \frac{693 \arctan\left(\frac{2 x - \sqrt{-2 + 2 \sqrt{3}}}{\sqrt{2 + 2 \sqrt{3}}}\right) \left(-2 + 2 \sqrt{3}\right) \sqrt{3}}{512 \sqrt{2 + 2 \sqrt{3}}} - \frac{3675 \arctan\left(\frac{2 x - \sqrt{-2 + 2 \sqrt{3}}}{\sqrt{2 + 2 \sqrt{3}}}\right) \left(-2 + 2 \sqrt{3}\right)}{512 \sqrt{2 + 2 \sqrt{3}}} + \frac{273 \arctan\left(\frac{2 x - \sqrt{-2 + 2 \sqrt{3}}}{\sqrt{2 + 2 \sqrt{3}}}\right) \sqrt{3}}{8 \sqrt{2 + 2 \sqrt{3}}}$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \frac{x^6 \left(5 x^6 + 3 x^4 + x^2 + 4\right)}{\left(x^4 + 2 x^2 + 3\right)^3} \, dx$$

Optimal(type 3, 179 leaves, 13 steps):

$$5x+\frac{25x(-x^2+3)}{16(x^4+2x^2+3)^2}+\frac{7x(58x^2+11)}{64(x^4+2x^2+3)}-\frac{\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-827621+1176531\sqrt{3}}}{512}$$
$$+\frac{\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-827621+1176531\sqrt{3}}}{512}+\frac{\arctan\left(\frac{-2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{827621+1176531\sqrt{3}}}{256}$$
$$-\frac{\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{827621+1176531\sqrt{3}}}{256}$$

Result(type 3, 421 leaves):

$$5x-\frac{-\frac{203}{32}x^7-\frac{889}{64}x^5-\frac{159}{8}x^3-\frac{531}{64}x}{(x^4+2x^2+3)^2}+\frac{943\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{1024}+\frac{185\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024}$$
$$-\frac{943\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)(-2+2\sqrt{3})\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}-\frac{185\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)(-2+2\sqrt{3})}{512\sqrt{2+2\sqrt{3}}}-\frac{379\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{3}}{64\sqrt{2+2\sqrt{3}}}$$
$$-\frac{943\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{1024}-\frac{185\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024}$$
$$-\frac{943\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)(-2+2\sqrt{3})\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}-\frac{185\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)(-2+2\sqrt{3})}{512\sqrt{2+2\sqrt{3}}}-\frac{379\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{3}}{64\sqrt{2+2\sqrt{3}}}$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int\frac{x^4(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^3}\,dx$$

Optimal(type 3, 174 leaves, 11 steps):

$$-\frac{25x(x^2+3)}{16(x^4+2x^2+3)^2}+\frac{x(-59x^2+238)}{64(x^4+2x^2+3)}-\frac{\arctan\left(\frac{-2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{-146505+98481\sqrt{3}}}{256}$$

$$+\frac{\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{-146505+98481\sqrt{3}}}{256}+\frac{\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{146505+98481\sqrt{3}}}{512}$$

$$-\frac{\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{146505+98481\sqrt{3}}}{512}$$

Result(type 3, 417 leaves):

$$\frac{-\frac{59}{64}x^7+\frac{15}{8}x^5+\frac{199}{64}x^3+\frac{207}{32}x}{(x^4+2x^2+3)^2}-\frac{307\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{1024}-\frac{399\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024}$$

$$+\frac{307\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}+\frac{399\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)}{512\sqrt{2+2\sqrt{3}}}-\frac{23\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{3}}{32\sqrt{2+2\sqrt{3}}}$$

$$+\frac{307\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{1024}+\frac{399\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024}$$

$$+\frac{307\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}+\frac{399\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)}{512\sqrt{2+2\sqrt{3}}}-\frac{23\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{3}}{32\sqrt{2+2\sqrt{3}}}$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int\frac{5x^6+3x^4+x^2+4}{(x^4+2x^2+3)^3}\,dx$$

Optimal(type 3, 176 leaves, 11 steps):

$$\frac{25x(-x^2+1)}{48(x^4+2x^2+3)^2}+\frac{x(51x^2+64)}{192(x^4+2x^2+3)}-\frac{\arctan\left(\frac{-2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{-3873+3057\sqrt{3}}}{768}+\frac{\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{-3873+3057\sqrt{3}}}{768}$$

$$+\frac{\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{3873+3057\sqrt{3}}}{1536}-\frac{\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{3873+3057\sqrt{3}}}{1536}$$

Result(type 3, 417 leaves):

$$\frac{\frac{17}{64}x^7+\frac{83}{96}x^5+\frac{181}{192}x^3+\frac{73}{48}x}{(x^4+2x^2+3)^2}-\frac{55\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{3072}-\frac{21\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024}$$

$$+\frac{55\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)\sqrt{3}}{1536\sqrt{2+2\sqrt{3}}}+\frac{21\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)}{512\sqrt{2+2\sqrt{3}}}-\frac{\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{3}}{48\sqrt{2+2\sqrt{3}}}$$

$$+\frac{55\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{3072}+\frac{21\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024}$$

$$+\frac{55\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)\sqrt{3}}{1536\sqrt{2+2\sqrt{3}}}+\frac{21\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)}{512\sqrt{2+2\sqrt{3}}}-\frac{\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{3}}{48\sqrt{2+2\sqrt{3}}}$$

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Problem 37: Result more than twice size of optimal antiderivative.
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$$\int\frac{5x^6+3x^4+x^2+4}{x^4\left(x^4+2x^2+3\right)^3}\,dx$$

```
Optimal(type 3, 186 leaves, 13 steps):
```

$$-\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x\left(5x^2+7\right)}{432\left(x^4+2x^2+3\right)^2}+\frac{x\left(1025x^2+1474\right)}{5184\left(x^4+2x^2+3\right)}+\frac{\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-30014223+33721353\sqrt{3}}}{124416}$$

$$-\frac{\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-30014223+33721353\sqrt{3}}}{124416}-\frac{\arctan\left(\frac{-2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{30014223+33721353\sqrt{3}}}{62208}$$

$$+\frac{\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{30014223+33721353\sqrt{3}}}{62208}$$

```
Result(type 3, 428 leaves):
```

$$-\frac{4}{81x^3}+\frac{7}{27x}+\frac{\frac{1025}{192}x^7+\frac{881}{48}x^5+\frac{7523}{192}x^3+\frac{1087}{32}x}{27\left(x^4+2x^2+3\right)^2}-\frac{4865\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{248832}$$

$$-\frac{127\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{82944}+\frac{4865\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)\sqrt{3}}{124416\sqrt{2+2\sqrt{3}}}$$

$$+\frac{127\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)}{41472\sqrt{2+2\sqrt{3}}}+\frac{1121\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{3}}{7776\sqrt{2+2\sqrt{3}}}$$

$$+\frac{4865\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{248832}+\frac{127\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{82944}$$

$$+\frac{4865\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)\sqrt{3}}{124416\sqrt{2+2\sqrt{3}}}+\frac{127\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\left(-2+2\sqrt{3}\right)}{41472\sqrt{2+2\sqrt{3}}}$$

$$+\frac{1121\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\sqrt{3}}{7776\sqrt{2+2\sqrt{3}}}$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int\frac{x^4\left(g\,x^6+f x^4+e x^2+d\right)}{\left(c x^4+b x^2+a\right)^2}\,dx$$

Optimal(type 3, 547 leaves, 6 steps):

$$\frac{\left(-2\,b\,g+c f\right)x}{c^3}+\frac{g\,x^3}{3\,c^2}$$

$$+\frac{x\left(a\left(2\,c^3 d-c^2\left(2\,a f+b\,e\right)-b^3 g+b\,c\left(3\,a\,g+b f\right)\right)+\left(b^3 c f+b\,c^2\left(-3\,a f+c\,d\right)-b^4 g-b^2 c\left(-4\,a\,g+c\,e\right)+2\,a\,c^2\left(-a\,g+c\,e\right)\right)x^2\right)}{2\,c^3\left(-4\,a\,c+b^2\right)\left(c x^4+b\,x^2+a\right)}$$

$$-\frac{1}{4\,c^{7/2}\left(-4\,a\,c+b^2\right)\sqrt{b-\sqrt{-4\,a\,c+b^2}}}\left(\arctan\left(\frac{x\sqrt{2}\,\sqrt{c}}{\sqrt{b-\sqrt{-4\,a\,c+b^2}}}\right)\left(3\,b^3 c f-b\,c^2\left(13\,a f+c\,d\right)-5\,b^4 g-b^2 c\left(-24\,a\,g+c\,e\right)+2\,a\,c^2\left(-7\,a\,g\right.\right.\right.$$
$$\left.\left.\left.+3\,c\,e\right)+\frac{-3\,b^4 c f+4\,a\,c^3\left(-5\,a f+c\,d\right)+b^2 c^2\left(19\,a f+c\,d\right)+5\,b^5 g+b^3 c\left(-34\,a\,g+c\,e\right)-4\,a\,b\,c^2\left(-13\,a\,g+2\,c\,e\right)}{\sqrt{-4\,a\,c+b^2}}\right)\sqrt{2}\right)$$

$$-\frac{1}{4\,c^{7/2}\left(-4\,a\,c+b^2\right)\sqrt{b+\sqrt{-4\,a\,c+b^2}}}\left(\arctan\left(\frac{x\sqrt{2}\,\sqrt{c}}{\sqrt{b+\sqrt{-4\,a\,c+b^2}}}\right)\left(3\,b^3 c f-b\,c^2\left(13\,a f+c\,d\right)-5\,b^4 g-b^2 c\left(-24\,a\,g+c\,e\right)+2\,a\,c^2\left(-7\,a\,g\right.\right.\right.$$
$$\left.\left.\left.+3\,c\,e\right)+\frac{3\,b^4 c f-4\,a\,c^3\left(-5\,a f+c\,d\right)-b^2 c^2\left(19\,a f+c\,d\right)-5\,b^5 g-b^3 c\left(-34\,a\,g+c\,e\right)+4\,a\,b\,c^2\left(-13\,a\,g+2\,c\,e\right)}{\sqrt{-4\,a\,c+b^2}}\right)\sqrt{2}\right)$$

Result(type ?, 8532 leaves): Display of huge result suppressed!

Test results for the 14 problems in "1.2.2.7 P(x) (d+e x^2)^q (a+b x^2+c x^4)^p.txt"

Problem 5: Unable to integrate problem.

$$\int \frac{(B x^2 + A)(e x^2 + d)^q}{c x^4 + a}\, dx$$

Optimal(type 6, 141 leaves, 6 steps):

$$\frac{x (e x^2 + d)^q \, AppellF1\left(\frac{1}{2}, -q, 1, \frac{3}{2}, -\frac{e x^2}{d}, -\frac{x^2 \sqrt{c}}{\sqrt{-a}}\right)\left(A - \frac{B\sqrt{-a}}{\sqrt{c}}\right)}{2 a \left(1 + \frac{e x^2}{d}\right)^q} + \frac{x (e x^2 + d)^q \, AppellF1\left(\frac{1}{2}, 1, -q, \frac{3}{2}, \frac{x^2 \sqrt{c}}{\sqrt{-a}}, -\frac{e x^2}{d}\right)\left(A + \frac{B\sqrt{-a}}{\sqrt{c}}\right)}{2 a \left(1 + \frac{e x^2}{d}\right)^q}$$

Result(type 8, 28 leaves):

$$\int \frac{(B x^2 + A)(e x^2 + d)^q}{c x^4 + a}\, dx$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \frac{(B x^2 + A)(e x^2 + d)^3}{\sqrt{c x^4 + b x^2 + a}}\, dx$$

Optimal(type 4, 739 leaves, 6 steps):

$$\frac{e\left(7 A c e (-4 b e + 15 c d) + B\left(105 c^2 d^2 + 24 b^2 e^2 - c e (25 a e + 84 b d)\right)\right) x \sqrt{c x^4 + b x^2 + a}}{105 c^3} + \frac{e^2 (7 A c e - 6 b B e + 21 B c d) x^3 \sqrt{c x^4 + b x^2 + a}}{35 c^2}$$

$$+ \frac{B e^3 x^5 \sqrt{c x^4 + b x^2 + a}}{7 c}$$

$$+ \frac{\left(7 A c e \left(45 c^2 d^2 + 8 b^2 e^2 - 3 c e (3 a e + 10 b d)\right) + B\left(105 c^3 d^3 - 48 b^3 e^3 - 21 c^2 d e (9 a e + 10 b d) + 8 b c e^2 (13 a e + 21 b d)\right)\right) x \sqrt{c x^4 + b x^2 + a}}{105 c^{7/2} \left(\sqrt{a} + x^2 \sqrt{c}\right)}$$

$$- \frac{1}{105 \cos\left(2 \arctan\left(\frac{c^{1/4} x}{a^{1/4}}\right)\right) c^{15/4} \sqrt{c x^4 + b x^2 + a}} \Bigg(a^{1/4} \left(7 A c e \left(45 c^2 d^2 + 8 b^2 e^2 - 3 c e (3 a e + 10 b d)\right) + B\left(105 c^3 d^3 - 48 b^3 e^3\right.\right.$$

$$\left.\left. - 21 c^2 d e (9 a e + 10 b d) + 8 b c e^2 (13 a e + 21 b d)\right)\right) \sqrt{\cos\left(2 \arctan\left(\frac{c^{1/4} x}{a^{1/4}}\right)\right)^2} \, \mathrm{EllipticE}\left(\sin\left(2 \arctan\left(\frac{c^{1/4} x}{a^{1/4}}\right)\right), \frac{\sqrt{2 - \frac{b}{\sqrt{a}\sqrt{c}}}}{2}\right) \left(\sqrt{a}\right.$$

$$+x^2\sqrt{c}\Big)\sqrt{\frac{c x^4+b x^2+a}{\left(\sqrt{a}+x^2\sqrt{c}\right)^2}}\Bigg)$$

$$+\frac{1}{210\cos\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)c^{15/4}\sqrt{c x^4+b x^2+a}}\Bigg(a^{1/4}\sqrt{\cos\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)^2}\,\mathrm{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\sqrt{c}}}}{2}\right)\Big(\sqrt{a}$$

$$+x^2\sqrt{c}\Big)\Big(7 A c e\left(45 c^2 d^2+8 b^2 e^2-3 c e\left(3 a e+10 b d\right)\right)+B\left(105 c^3 d^3-48 b^3 e^3-21 c^2 d e\left(9 a e+10 b d\right)+8 b c e^2\left(13 a e+21 b d\right)\right)$$

$$+\frac{\left(7 A c\left(4 a b e^3-15 a c d e^2+15 c^2 d^3\right)-a B e\left(105 c^2 d^2+24 b^2 e^2-c e\left(25 a e+84 b d\right)\right)\right)\sqrt{c}}{\sqrt{a}}\Big)\sqrt{\frac{c x^4+b x^2+a}{\left(\sqrt{a}+x^2\sqrt{c}\right)^2}}\Bigg)$$

Result(type 4, 1707 leaves):

$$\frac{1}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c x^4+b x^2+a}}\Bigg(A d^3\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\Bigg(\frac{1}{2}\Big(x\sqrt{2}$$

$$\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\Big),\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\Bigg)\Bigg)+\left(A e^3+3 B d e^2\right)\Bigg(\frac{x^3\sqrt{c x^4+b x^2+a}}{5c}-\frac{4 b x\sqrt{c x^4+b x^2+a}}{15c^2}$$

$$+\frac{1}{15c^2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c x^4+b x^2+a}}\Bigg(b a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\Bigg(\frac{1}{2}\Big(x$$

$$\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\Big),\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\Bigg)\Bigg)-\frac{1}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c x^4+b x^2+a}\left(b+\sqrt{-4ac+b^2}\right)}\Bigg(\Big(-\frac{3a}{5c}$$

$$+\frac{8b^2}{15c^2}\Bigg) a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\right.\right.$$

$$\left.\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)\Bigg)\Bigg)\Bigg)+\big(3Ade^2$$

$$+3Bd^2e\big)\left(\frac{x\sqrt{cx^4+bx^2+a}}{3c}\right.$$

$$-\frac{1}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}\left(a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(x\right.\right.\right.$$

$$\left.\left.\left.\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\right),\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)\right)$$

$$+\frac{1}{3c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}\left(b+\sqrt{-4ac+b^2}\right)}\left(ba\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\right.$$

$$\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\right.\right.$$

$$\left.\left.\left.\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)\right)\right)-\frac{1}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}\left(b+\sqrt{-4ac+b^2}\right)}\Big(\big(3Ad^2e$$

$$+Bd^3\Big)\,a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\right.\right.$$

$$\left.\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)-\text{EllipticE}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)\Bigg)\Bigg)\Bigg)+Be^3\left(\frac{x^5\sqrt{cx^4+bx^2+a}}{7c}\right.$$

$$-\frac{6bx^3\sqrt{cx^4+bx^2+a}}{35c^2}+\frac{\left(-\frac{5a}{7c}+\frac{24b^2}{35c^2}\right)x\sqrt{cx^4+bx^2+a}}{3c}-\frac{1}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}\left(\left(-\frac{5a}{7c}\right.\right.$$

$$\left.+\frac{24b^2}{35c^2}\right)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\right.$$

$$\left.\left.\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)\right)-\frac{1}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}\left(b+\sqrt{-4ac+b^2}\right)}\left(\left(\frac{18ba}{35c^2}\right.\right.$$

$$\left.-\frac{2\left(-\frac{5a}{7c}+\frac{24b^2}{35c^2}\right)b}{3c}\right)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\right.\right.$$

$$\left.\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)-\text{EllipticE}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)\Bigg)\Bigg)\Bigg)$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \frac{(Bx^2+A)(ex^2+d)^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

Optimal(type 4, 516 leaves, 5 steps):

$$\frac{e(5Ace-4bBe+10Bcd)x\sqrt{cx^4+bx^2+a}}{15c^2}+\frac{Be^2x^3\sqrt{cx^4+bx^2+a}}{5c}$$
$$+\frac{\left(10Ace(-be+3cd)+B\left(15c^2d^2+8b^2e^2-ce(9ae+20bd)\right)\right)x\sqrt{cx^4+bx^2+a}}{15c^{5/2}\left(\sqrt{a}+x^2\sqrt{c}\right)}$$
$$-\frac{1}{15\cos\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)c^{11/4}\sqrt{cx^4+bx^2+a}}\Bigg(a^{1/4}\left(10Ace(-be+3cd)+B\left(15c^2d^2+8b^2e^2-ce(9ae+20bd)\right)\right)\sqrt{\cos\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)^2}\,\mathrm{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\sqrt{c}}}}{2}\right)\left(\sqrt{a}+x^2\sqrt{c}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{a}+x^2\sqrt{c}\right)^2}}\Bigg)$$
$$+\frac{1}{30\cos\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)c^{11/4}\sqrt{cx^4+bx^2+a}}\Bigg(a^{1/4}\sqrt{\cos\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)^2}\,\mathrm{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\sqrt{c}}}}{2}\right)\left(\sqrt{a}+x^2\sqrt{c}\right)\left(10Ace(-be+3cd)+B\left(15c^2d^2+8b^2e^2-ce(9ae+20bd)\right)-\frac{\left(2aBe(-2be+5cd)-5Ac\left(-ae^2+3cd^2\right)\right)\sqrt{c}}{\sqrt{a}}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{a}+x^2\sqrt{c}\right)^2}}\Bigg)$$

Result(type 4, 1200 leaves):

$$\frac{1}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}\Bigg(Ad^2\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\Bigg(\frac{1}{2}\Big(x\sqrt{2}$$

$$\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\Bigg), \sqrt{\frac{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}{2}}\Bigg)\Bigg) + (Ae^2+2Bde)\Bigg(\frac{x\sqrt{cx^4+bx^2+a}}{3c}$$

$$-\frac{1}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}\Bigg(a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\Bigg(\frac{1}{2}\Bigg(x$$

$$\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\Bigg), \sqrt{\frac{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}{2}}\Bigg)\Bigg)$$

$$+\frac{1}{3c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}\left(b+\sqrt{-4ac+b^2}\right)}\Bigg(ba\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}$$

$$\Bigg(\mathrm{EllipticF}\Bigg(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}, \frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\Bigg) - \mathrm{EllipticE}\Bigg(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},$$

$$\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\Bigg)\Bigg)\Bigg)\Bigg) - \frac{1}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}\left(b+\sqrt{-4ac+b^2}\right)}\Bigg((2Ade$$

$$+Bd^2)\,a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\Bigg(\mathrm{EllipticF}\Bigg(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},$$

$$\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\Bigg) - \mathrm{EllipticE}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}, \frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)\Bigg)\Bigg) + B e^2 \Bigg(\frac{x^3\sqrt{cx^4+bx^2+a}}{5c}$$

$$-\frac{4bx\sqrt{cx^4+bx^2+a}}{15c^2}$$

$$+\frac{1}{15c^2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}\Bigg(ba\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\Bigg(\frac{1}{2}\Bigg(x$$

$$\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\Bigg), \frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\Bigg)\Bigg) - \frac{1}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}\left(b+\sqrt{-4ac+b^2}\right)}\Bigg(\Bigg(-\frac{3a}{5c}$$

$$+\frac{8b^2}{15c^2}\Bigg)a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\Bigg(\mathrm{EllipticF}\Bigg(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},$$

$$\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\Bigg) - \mathrm{EllipticE}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}, \frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)\Bigg)\Bigg)\Bigg)$$

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Problem 9: Result more than twice size of optimal antiderivative.
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$$\int \frac{(Bx^2+A)(ex^2+d)}{\sqrt{cx^4+bx^2+a}}\,dx$$

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Optimal(type 4, 360 leaves, 4 steps):
```

$$\frac{B e x\sqrt{c x^4+b x^2+a}}{3 c}+\frac{(3 A c e-2 b B e+3 B c d) x\sqrt{c x^4+b x^2+a}}{3 c^{3/2}\left(\sqrt{a}+x^2\sqrt{c}\right)}$$

$$-\frac{1}{3\cos\left(2\arctan\left(\frac{c^{1/4} x}{a^{1/4}}\right)\right) c^{7/4}\sqrt{c x^4+b x^2+a}}\Bigg(a^{1/4}(3 A c e-2 b B e$$

$$+3 B c d)\sqrt{\cos\left(2\arctan\left(\frac{c^{1/4} x}{a^{1/4}}\right)\right)^2}\,\mathrm{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{1/4} x}{a^{1/4}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\sqrt{c}}}}{2}\right)\left(\sqrt{a}+x^2\sqrt{c}\right)\sqrt{\frac{c x^4+b x^2+a}{\left(\sqrt{a}+x^2\sqrt{c}\right)^2}}\Bigg)$$

$$+\frac{1}{6\cos\left(2\arctan\left(\frac{c^{1/4} x}{a^{1/4}}\right)\right) c^{7/4}\sqrt{c x^4+b x^2+a}}\Bigg(a^{1/4}\sqrt{\cos\left(2\arctan\left(\frac{c^{1/4} x}{a^{1/4}}\right)\right)^2}\,\mathrm{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{1/4} x}{a^{1/4}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\sqrt{c}}}}{2}\right)\Big(\sqrt{a}$$

$$+x^2\sqrt{c}\Big)\left(3 B c d-2 b B e+3 A c e+\frac{(3 A c d-a B e)\sqrt{c}}{\sqrt{a}}\right)\sqrt{\frac{c x^4+b x^2+a}{\left(\sqrt{a}+x^2\sqrt{c}\right)^2}}\Bigg)$$

Result(type 4, 758 leaves):

$$\frac{1}{4\sqrt{\frac{-b+\sqrt{-4 a c+b^2}}{a}}\sqrt{c x^4+b x^2+a}}\Bigg(A d\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4 a c+b^2}\right) x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4 a c+b^2}\right) x^2}{a}}\,\mathrm{EllipticF}\Bigg(\frac{1}{2}\Big(x\sqrt{2}$$

$$\sqrt{\frac{-b+\sqrt{-4 a c+b^2}}{a}}\Big),\frac{\sqrt{-4+\frac{2 b\left(b+\sqrt{-4 a c+b^2}\right)}{a c}}}{2}\Bigg)\Bigg)-\frac{1}{2\sqrt{\frac{-b+\sqrt{-4 a c+b^2}}{a}}\sqrt{c x^4+b x^2+a}\left(b+\sqrt{-4 a c+b^2}\right)}\Bigg((A e$$

$$+B d) a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4 a c+b^2}\right) x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4 a c+b^2}\right) x^2}{a}}\Bigg(\mathrm{EllipticF}\Bigg(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4 a c+b^2}}{a}}}{2},$$

$$\left.\left.\left.\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)\right)\right)+Be\left(\frac{x\sqrt{cx^4+bx^2+a}}{3c}\right.$$

$$-\frac{1}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}\left(a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(x\right.\right.\right.$$

$$\left.\left.\left.\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\right),\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)\right)$$

$$+\frac{1}{3c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}\left(b+\sqrt{-4ac+b^2}\right)}\left(ba\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\right.$$

$$\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\right.\right.$$

$$\left.\left.\left.\left.\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)\right)\right)\right)$$

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Problem 10: Result more than twice size of optimal antiderivative.
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$$\int\frac{Bx^2+A}{\left(ex^2+d\right)^3\sqrt{cx^4+bx^2+a}}\,dx$$

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Optimal(type 4, 1080 leaves, 7 steps):
```

$$-\frac{1}{16d^{5/2}\left(ae^2-bde+cd^2\right)^{5/2}\sqrt{e}}\Big(\big(Bd\left(3c^2d^4-10acd^2e^2+ae^3\left(-ae+4bd\right)\right)-Ae\left(15c^2d^4-2cd^2e\left(-3ae+10bd\right)+e^2\left(3a^2e^2-8abde\right.\right.$$

$$+8b^2d^2)))\arctan\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\,\sqrt{e}\,\sqrt{cx^4+bx^2+a}}\right)\Bigg)-\frac{e(-Ae+Bd)x\sqrt{cx^4+bx^2+a}}{4d(ae^2-bde+cd^2)(ex^2+d)^2}$$

$$+\frac{e(3Ae(3cd^2-e(-ae+2bd))-Bd(5cd^2-e(ae+2bd)))x\sqrt{cx^4+bx^2+a}}{8d^2(ae^2-bde+cd^2)^2(ex^2+d)}$$

$$-\frac{(3Ae(3cd^2-e(-ae+2bd))-Bd(5cd^2-e(ae+2bd)))x\sqrt{c}\,\sqrt{cx^4+bx^2+a}}{8d^2(ae^2-bde+cd^2)^2(\sqrt{a}+x^2\sqrt{c})}$$

$$+\frac{1}{8\cos\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)d^2(ae^2-bde+cd^2)^2\sqrt{cx^4+bx^2+a}}\Bigg(a^{1/4}c^{1/4}(3Ae(3cd^2-e(-ae+2bd))-Bd(5cd^2-e(ae$$

$$+2bd)))\sqrt{\cos\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)^2}\,\mathrm{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\,\sqrt{c}}}}{2}\right)(\sqrt{a}+x^2\sqrt{c})\sqrt{\frac{cx^4+bx^2+a}{(\sqrt{a}+x^2\sqrt{c})^2}}\Bigg)$$

$$+\Bigg((Bd(3c^2d^4-10acd^2e^2+ae^3(-ae+4bd))-Ae(15c^2d^4-2cd^2e(-3ae+10bd)+e^2(3a^2e^2-8abde+8b^2d^2)))\sqrt{\cos\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)^2}$$

$$+d\sqrt{c})\sqrt{cx^4+bx^2+a}\Bigg)+\Bigg(c^{1/4}\sqrt{\cos\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)^2}\,\mathrm{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\,\sqrt{c}}}}{2}\right)(\sqrt{a}$$

$$+x^2\sqrt{c})(ae(3Ae+Bd)+4Ad(-be+cd)+d(-Ae+Bd)\sqrt{a}\,\sqrt{c})\sqrt{\frac{cx^4+bx^2+a}{(\sqrt{a}+x^2\sqrt{c})^2}}\Bigg)\Bigg/\Bigg(8\cos\left(2\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)a^{1/4}d^2(ae^2$$

$$-bde+cd^2)(-e\sqrt{a}+d\sqrt{c})\sqrt{cx^4+bx^2+a}\Bigg)$$

Result(type ?, 4475 leaves): Display of huge result suppressed!

Problem 11: Result more than twice size of optimal antiderivative.

$$\int\frac{Bx^2+A}{(ex^2+d)^2(cx^4+bx^2+a)^{3/2}}\,dx$$

Optimal(type 4, 1252 leaves, 15 steps):

$$\frac{e^{3/2}\left(A e\left(7 c d^2-e\left(-a e+4 b d\right)\right)-B d\left(5 c d^2-e\left(a e+2 b d\right)\right)\right) \arctan\left(\frac{x\sqrt{a e^2-b d e+c d^2}}{\sqrt{d}\,\sqrt{e}\,\sqrt{c x^4+b x^2+a}}\right)}{4 d^{3/2}\left(a e^2-b d e+c d^2\right)^{5/2}}$$

$$+\frac{1}{a\left(-4 a c+b^2\right)\left(a e^2-b d e+c d^2\right)^2\sqrt{c x^4+b x^2+a}}\left(x\left(a b c\left(A e\left(-b e+2 c d\right)-B\left(-a e^2+c d^2\right)\right)+\left(-2 a c+b^2\right)\left(a B e\left(-b e+2 c d\right)+A\left(c^2 d^2\right.\right.\right.\right.$$

$$\left.\left.\left.\left.+b^2 e^2-c e\left(a e+2 b d\right)\right)\right)-c\left(a B\left(2 c^2 d^2+b^2 e^2-2 c e\left(a e+b d\right)\right)+A\left(2 b^2 c d e-4 a c^2 d e-b^3 e^2-b c\left(-3 a e^2+c d^2\right)\right)\right) x^2\right)\right)$$

$$-\frac{e^3\left(-A e+B d\right) x\sqrt{c x^4+b x^2+a}}{2 d\left(a e^2-b d e+c d^2\right)^2\left(e x^2+d\right)}$$

$$+\frac{1}{2 a\left(4 a c-b^2\right) d\left(c d^2+e\left(a e-b d\right)\right)^2\left(\sqrt{a}+x^2\sqrt{c}\right)}\left(\left(a B d\left(-4 c^2 d^2-3 b^2 e^2+4 c e\left(2 a e+b d\right)\right)+A\left(2 b^3 d e^2+2 b c d\left(-3 a e^2+c d^2\right)\right.\right.\right.$$

$$\left.\left.\left.-4 a c e\left(a e^2-2 c d^2\right)+b^2\left(a e^3-4 d^2 e c\right)\right)\right) x\sqrt{c}\,\sqrt{c x^4+b x^2+a}\right)-\left(c^{1/4}\left(a B d\left(4 c^2 d^2+3 b^2 e^2-4 c e\left(2 a e+b d\right)\right)-A\left(2 b^3 d e^2\right.\right.\right.$$

$$\left.\left.+2 b c d\left(-3 a e^2+c d^2\right)-4 a c e\left(a e^2-2 c d^2\right)+b^2\left(a e^3-4 d^2 e c\right)\right)\right)\sqrt{\cos\left(2\arctan\left(\frac{c^{1/4} x}{a^{1/4}}\right)\right)^2}\ \mathrm{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{1/4} x}{a^{1/4}}\right)\right),\right.$$

$$\left.\frac{\sqrt{2-\frac{b}{\sqrt{a}\,\sqrt{c}}}}{2}\right)\left(\sqrt{a}+x^2\sqrt{c}\right)\sqrt{\frac{c x^4+b x^2+a}{\left(\sqrt{a}+x^2\sqrt{c}\right)^2}}\Bigg)\Bigg/\left(2\cos\left(2\arctan\left(\frac{c^{1/4} x}{a^{1/4}}\right)\right) a^{3/4}\left(-4 a c+b^2\right) d\left(c d^2+e\left(a e\right.\right.\right.$$

$$\left.\left.\left.-b d\right)\right)^2\sqrt{c x^4+b x^2+a}\right)-\left(e\left(A e\left(7 c d^2-e\left(-a e+4 b d\right)\right)-B d\left(5 c d^2-e\left(a e\right.\right.\right.\right.\right.$$

$$\left.\left.\left.+2 b d\right)\right)\right)\sqrt{\cos\left(2\arctan\left(\frac{c^{1/4} x}{a^{1/4}}\right)\right)^2}\ \mathrm{EllipticPi}\left(\sin\left(2\arctan\left(\frac{c^{1/4} x}{a^{1/4}}\right)\right),-\frac{\left(-e\sqrt{a}+d\sqrt{c}\right)^2}{4 d e\sqrt{a}\,\sqrt{c}},\frac{\sqrt{2-\frac{b}{\sqrt{a}\,\sqrt{c}}}}{2}\right)\left(e\sqrt{a}+d\sqrt{c}\right)\left(\sqrt{a}\right.$$

$$\left.+x^2\sqrt{c}\right)\sqrt{\frac{c x^4+b x^2+a}{\left(\sqrt{a}+x^2\sqrt{c}\right)^2}}\Bigg)\Bigg/\left(8\cos\left(2\arctan\left(\frac{c^{1/4} x}{a^{1/4}}\right)\right) a^{1/4} c^{1/4} d^2\left(a e^2-b d e+c d^2\right)^2\left(-e\sqrt{a}+d\sqrt{c}\right)\sqrt{c x^4+b x^2+a}\right)$$

$$\left.\right)\left(e\sqrt{a}-d\sqrt{c}\right)\left(-2\sqrt{a}\,\sqrt{c}+b\right)\sqrt{c x^4+b x^2+a}\Big)$$

Result(type ?, 8275 leaves): Display of huge result suppressed!

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c x^4 + b x^2 + a}}{-c d x^4 + a d}\, dx$$

Optimal(type 3, 105 leaves, 4 steps):

$$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{-2\sqrt{a}\,\sqrt{c}+b}}{\sqrt{c x^4+b x^2+a}}\right)\sqrt{-2\sqrt{a}\,\sqrt{c}+b}}{4 d\sqrt{a}\,\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2\sqrt{a}\,\sqrt{c}+b}}{\sqrt{c x^4+b x^2+a}}\right)\sqrt{2\sqrt{a}\,\sqrt{c}+b}}{4 d\sqrt{a}\,\sqrt{c}}$$

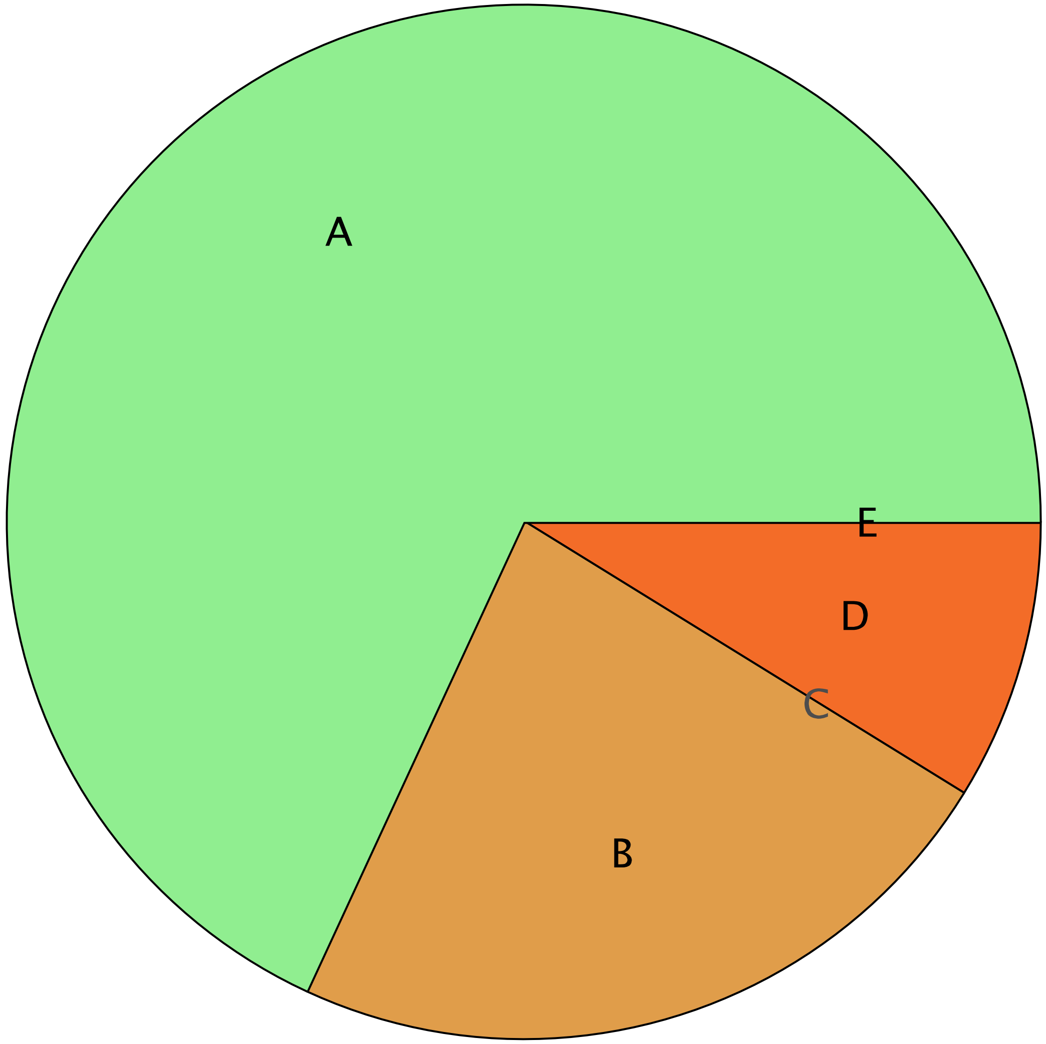
Result(type 3, 237 leaves):

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{c x^4+b x^2+a}\,\sqrt{2}}{x\sqrt{-4\sqrt{a c}-2 b}}\right)}{2 d\sqrt{-4\sqrt{a c}-2 b}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{c x^4+b x^2+a}\,\sqrt{2}}{x\sqrt{-4\sqrt{a c}-2 b}}\right) b}{4 d\sqrt{a c}\,\sqrt{-4\sqrt{a c}-2 b}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{c x^4+b x^2+a}\,\sqrt{2}}{x\sqrt{4\sqrt{a c}-2 b}}\right)}{2 d\sqrt{4\sqrt{a c}-2 b}}$$
$$+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{c x^4+b x^2+a}\,\sqrt{2}}{x\sqrt{4\sqrt{a c}-2 b}}\right) b}{4 d\sqrt{a c}\,\sqrt{4\sqrt{a c}-2 b}}$$

Test results for the 2 problems in "1.2.2.8 P(x) (d+e x)^q (a+b x^2+c x^4)^p.txt"

Summary of Integration Test Results

602 integration problems



A - 410 optimal antiderivatives
B - 139 more than twice size of optimal antiderivatives
C - 0 unnecessarily complex antiderivatives
D - 53 unable to integrate problems
E - 0 integration timeouts